TEZPUR UNIVERSITY
Assignment (Spring 2020)
MMS 402: FunctionalAnalysis

Total Marks: 30

The figures in the right-hand margin indicate marks for the individual question.
All questions are compulsory.
Answers should be concise and entire answer to a question should be together. State assumptions wherever made.

1. Let X, Y be normed linear spaces and $T : X \to Y$ be bounded linear. We know that

$$\|T\| = sup_{0 \neq x \in X} \frac{\|Tx\|}{\|x\|} = inf\{c > 0 : \|Tx\| \leq c\|x\| \,\forall\, x \in X\}.$$

If $\alpha := sup_{\|x\| \leq 1} \|Tx\|,\ \beta := sup_{\|x\|=1} \|Tx\|,\ \gamma := sup_{\|x\|<1} \|Tx\|$, then show that $\alpha = \beta = \gamma = \|T\|$. 5

2. Let X and Y be two normed linear spaces, and $B(X, Y)$ denote the space of all bounded linear operators from X to Y. Show that $B(X, Y)$ is a Banach space if and only if Y is a Banach space. 6+6=12
(*It may be assumed that $B(X, Y)$ is a normed linear space.*)

3. Let $S, T : C[0, 1] \to C[0, 1]$ be defined as $(Sx)s = s \int_0^1 x(t)dt$ and $(Tx)s = sx(s)$ for $x \in C[0, 1]$ and $s \in [0, 1]$. Show that $ST \neq TS$ and find $\|ST\|$, $\|TS\|$. 2+3+3=8

4. Let X be a normed linear space and X', X'' denote the dual and the second dual of X respectively. For $x \in X$ define $\varphi_x : X' \to \mathbb{F}$ as $\varphi_x(f) = f(x)$. Show that $\varphi_x \in X''$. 3

5. If $\varphi : X \to X''$ is defined as $\varphi(x) = \varphi_x$, where $\varphi_x(f) = f(x) \forall f \in X''$, then show that φ is an isometry. 2

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