TU/CODL/MATH

Centre for Distance and Open Learning
TEZPUR UNIVERSITY
Assignment (Spring 2020)

MMS101: Abstract Algebra

Total Marks : 30

---

1. Let G be an abelian group. Let $x, y \in G$ such that $o(x) = 5$ and $o(y) = 7$ then find the order of $(xy)^{14}$. 3

2. Show that any quotient group of an abelian group is abelian. 4

3. Suppose that H_1 and H_2 are two subgroups of a group G. If $|H_1| = 36$ and $|H_2| = 70$ then find all the possible values of $|H_1 \cap H_2|$. 4

4. Let G be a group and H a non empty subset of G. Prove that H is a subgroup of G if and only if $ab^{-1} \in H$ for all $a, b \in H$. 4

5. Let α and β be two elements of S_8 such that
$$\alpha = \left(\begin{smallmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 5 & 1 & 6 & 4 & 2 & 7 & 3 \end{smallmatrix}\right) \text{ and } \beta = \left(\begin{smallmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 4 & 8 & 1 & 7 & 2 & 5 & 3 \end{smallmatrix}\right).$$
Find $\beta\alpha\beta^{-1}\alpha^{-1}$. Also find the order of $\beta\alpha\beta^{-1}\alpha^{-1}$. 3+2=5

6. Let $f\colon R \to S$ be a ring homomorphism.
   (a) What is $\mathrm{Ker} f$? 1
   (b) Show that $\mathrm{Ker} f$ is an ideal of R. 2
   (c) Show that any ideal of R is a kernel of some ring homomorphism. 2

7. Let A_n and B_n denote the set of even and odd permutations of S_n, for $n \geq 2$, respectively. Show that $|A_n| = |B_n|$. Also find the order of B_n. 5

*****