TU/CODL/MMS/SPR/2018

TEZPUR UNIVERSITY

Assignment (Spring) 2018

MMS 402 [redacted] **Functional Analysis**

Total Marks: 30

The figures in the right-hand margin indicate marks for the individual question.
All questions are compulsory.
Answers should be concise and entire answer to a question should be together. State assumptions wherever made.

1. Show that a bounded metric on a non trivial linear space can never be induced by a norm. 5

2. Let d be a metric on Ω, and d_1 be another metric on Ω defined as $d_1(x,y) = \frac{d(x,y)}{1+d(x,y)}$. Show that d_1 cannot be induced by a norm. 5

3. For $1 \leq p \leq \infty$, show that the map $\nu(x) = |x(a)| + \|x'\|_p$ is a norm on $C^1[a,b]$. Also show that the map $\eta(x) = \|x'\|_p$ is not a norm on $C^1[a,b]$. 5

4. Let X be a normed linear space and X_0 be a closed subspace of X. Show that $\|[x]\| = dist(x, X_0)$ defines a norm on the quotient space X/X_0. Here $dist(x, X_0) = inf\{\|x-y\| : y \in X_0\}$. 5

5. Let X be an inner product space with $\|x\| := \langle x, x\rangle^{1/2}\, \forall x \in X$. Show that $|\langle x, y\rangle| \leq \|x\|\,\|y\|\, \forall x, y \in X$. 3

6. Show that l^p cannot be an inner product space unless $p = 2$. 3

7. Let X be an inner product space and $x, y \in X$. If $\|kx+y\|^2 = \|kx\|^2 + \|y\|^2\ \ \forall\ k \in \mathbb{F}$, then show that $x \perp y$. 4

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