

TEZPUR UNIVERSITY
Assignment (Spring) 2018
MMS304: Advanced Analysis

Total Marks: 30

Each question carries 6 marks.

All questions are compulsory.

Answers should be concise and entire answer to a question should be together. State assumptions wherever made.

1. If \mathcal{J} is the class of all closed intervals and \mathcal{B}_* is the σ -algebra generated by \mathcal{J} , then prove that $\mathcal{B}_* = \mathcal{B}$, where \mathcal{B} is the Borel σ -algebra.
2. If $m^*(A) = 0$ then prove that $m^*(A \cup B) = m^*(B)$ and $m^*(B - A) = m^*(B)$ for any subset B of \mathbb{R} .
3. If f and g are measurable and $g(x) \neq 0$ on the domain D then prove that $\frac{f}{g}$ is also measurable.
4. If f is a bounded measurable function defined on a set E of finite measure and A, B are disjoint measurable subsets of E then prove that

$$\int_{A \cup B} f = \int_A f + \int_B f.$$

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5. Prove that a measurable set B is negative with respect to a signed measure ν if $\nu(B \cap E) \leq 0$, for any measurable set E .

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