ASSIGNMENT MMS 202

Total marks: 30

The students may attempt any 6 (six) questions.

All questions carry equal marks.

1. On the three point-set $X = \{a, b, c, d\}$, find 5 topologies having more than 5 and less than 8 members.

2. Let X be an infinite set and let $p \in X$. Let $\mathscr{T} = \{A \subseteq X : X - A$ is either finite or contains $p\}$. Verify whether $\mathscr{T}$ is a topology on X.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = x + 2$. Determine if f is continuous.

4. Let X be a topological space. Let $\triangle$ denote the set $\{(x, x) : x \in X\}$ which is a subset of $X \times X$. If A, B are two subsets of X, show that $(A \times B) \cap \triangle = \phi$ iff $A \cap B = \phi$.

5. On $\mathbb{R}$, consider the equivalence relation defined by $x \sim y$ if $x - y \in \mathbb{Z}$. Describe the quotient space that results from the partition of $\mathbb{R}$ into the equivalent classes in the equivalence relation.

6. Let X, Y be two topological spaces. Let $x_0 \in X$. Show that $x_0 \times Y$ is homeomorphic to Y.

7. Examine the space $\mathbb{R}_\ell$ (Real line with Lower limit topology) for the following:
   (a) Separability (b) First Countability (c) Second Countability

8. Show that if X is compact and $f : X \to \mathbb{R}$ is continuous, then there exists $a, b \in X$ such that $f(x) \leq f(x) \leq f(b)$, for every $x \in X$.

9. Let $p : X \to Y$ be a closed continuous surjective map, where Y is compact, such that $p^{-1}(\{y\})$ is compact, for each $y \in Y$. Show that X is compact.

10. Find out which of the following are connected:
    (a) $\mathbb{R}_\ell$ (The real line with lower limit topology)
    (b) X a two point space with indiscrete topology.
    (c) The set $\mathbb{Q}$ of rationals in $\mathbb{R}$
    (d) The subset $X = \{x \times y : y = 0\} \cup \{x \times y : x > 0 \; and \; y = \frac{1}{x}\}$ as a subset of $\mathbb{R}^2$