TU/CODL/MMS/SPR/2018

TEZPUR UNIVERSITY
Assignment (Spring) 2018
MMS103: Real Analysis

Total Marks: 30

The figures in the right-hand margin indicate marks for the individual question.
All questions are compulsory.
Answers should be concise and entire answer to a question should be together. State assumptions wherever made.

1. Show that the following are equivalent. **9**
   (a) A is countable
   (b) There exists a injection from A to $\mathbb{N}$.
   (c) There exists a surjection from $\mathbb{N}$ to A.

2. Show that a subset of a metric space is closed if and only if its complement is open. **6**

3. In a discrete metric space (X, d), prove that a sequence $\{x_n\}$ is convergent iff it is eventually constant. **5**

4. Let (X, d_1) and (Y, d_2) be two metric spaces and $f : X \to Y$ be a function. Then, the following statements are equivalent: **10**
   (a) f is continuous.
   (b) If $\{x_n\}$ converges to $x \in X$ then $\{f(x_n)\}$ converges to $f(x)$ in Y.
   (c) $f^{-1}(G)$ is open in X whenever G is open in Y.

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