TEZPUR UNIVERSITY
Assignment (Spring) 2018
MMS101: Abstract Algebra

Total Marks: 30

The figures in the right-hand margin indicate marks for the individual question.
All questions are compulsory.
Answers should be concise and entire answer to a question should be together. State assumptions wherever made.

1. Let G be a group and $a \in G$. Let $Z(G)$ denotes the center of G and $C_G(a)$ denotes the centralizer of a in G. Then show that **3+3+3=9**
   (a) $Z(G)$ is a subgroup of G.
   (b) $C_G(a)$ is a subgroup of G.
   (c) $Z(G)$ is equal to $\bigcap_{a \in G} C_G(a)$.
2. Consider the symmetric group of degree 3 denoted by S_3. **4+2=6**
   (a) Determine all the centralizers in S_3.
   (b) Using 1(c), find the center of S_3.
3. Let $S = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} : a, b \in \mathbb{Z} \right\}$. **4+2=6**
   (a) Show that S is a subring of $M_2(\mathbb{Z})$.
   (b) Does unity exist in S?
4. Answer the following questions. **3+3+3=9**
   (a) Define Euclidean domain, principal ideal domain and unique factorization domain.
   (b) Show that every Euclidean domain is a principal ideal domain.
   (c) Give an example of a principal ideal domain but not an Euclidean domain.

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