

**TEZPUR UNIVERSITY**  
**Assignment Spring 2022**  
**MMS 401: Mathematical Methods**  
**Total Marks: 30**

*The figures in the right-hand margin indicate marks for the individual question.*

*All questions are compulsory.*

*Answers should be concise and entire answer to a question should be together. State assumptions wherever made.*

1. Convert the differential equation  $\frac{d^2\phi}{dx^2} + x\phi = 1$  with boundary condition  $\phi(0) = 0, \phi(1) = 1$  to its corresponding Fredholm integral equation of second kind and also recover the boundary value problem from the integral equation that you obtain. **7**
2. Show that  $\int x^m J_n(x) dx = \frac{x^{m+2} J_{n+1}}{(m+1-n)(m+1+n)} + \frac{x^{m+1} J_n}{(m+n+1)} + \frac{\int x^{m+2} J_n dx}{(m+1-n)(m+1+n)}$  and evaluate  $\int x^{-2} J_4(x) dx$ . **7**
3. Show  $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$  for  $n \geq 1$ . **5**
4. Show that Legendre polynomials are orthogonal. **5**
5. Find the Green's function for damped harmonic oscillator given by the equation  $m \frac{d^2 y}{dt^2} + c \frac{dy}{dt} + ky = f(t)$ , with  $m$  mass of the spring,  $c$  damping coefficient,  $k$  spring constant and  $f(t)$  force. **6**

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