TEZPUR UNIVERSITY

MMS 302: Partial Differential Equations

Total Marks: 30

The figures in the right-hand margin indicate marks for the individual question.
All questions are compulsory.
Answers should be concise and entire answer to a question should be together. State assumptions wherever made.

1. Define a quasi-linear PDE. Is $(\cos u + u_x^2)u_{xx} + (1 + u_y^2)u_{yy} = 0$ quasi-linear? [2]

2. Is $u(x,t) = \frac{1}{\sqrt{4\pi kt}} e^{-x^2/4kt}, k > 0$ a solution to the heat equation $u_t = ku_{xx}$ for $t > 0$ and $x \in \mathbb{R}$? [3]

3. Does the PDE $u_{xy} + u_{yy} + 2u_x + 3u_y = 0$ have solution along $x = 5$ and $x = y$? [3]

4. Is the equation $u_{xx}u_{yy} - u_{xy}^2 = 3x^2$ elliptic ? [2]

5. Reduce to Canonical form $y^2u_{xx} - 2yu_{xy} + u_{yy} - u_y - 8y = 0$. [4]

6. Use Riemann-Green function method to find the the solution of the PDE [4]

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{x+y}$$

such that $z = 0, p = \frac{2y}{x+y}$ along the line $x = y$.

7. Use the method of separation of variables to find the solution of the problem [5]

$$\begin{aligned} u_{tt} &= u_{xx}, \quad 0 < x < \pi, \quad t > 0, \\ u(x,0) &= \sin 2x, \quad u_t(x,0) = 0, \quad 0 \le x \le \pi, \\ u(0,t) &= 0 = u(\pi,t) \quad \text{for} \quad t > 0. \end{aligned}$$

8. Find the Fourier transform of Dirac delta. [2]

9. Is the function $f(t) = \cos t - e^{-(2t+1)^2}$ of exponential order? [2]

10. Use shifting theorem to find the Laplace transform of [3]

$$f(t) = \begin{cases} 0 & \text{if } 0 < t < 2\,, \\ t-1 & \text{if } t > 2. \end{cases}$$