ASSIGNMENTS
MMS 103 :: Real Analysis

Unless or otherwise explicitly stated the metric in $\mathbb{R}$ will be considered as the usual metric in the following questions.

(1) Define countable set. Give two examples of countable and two examples of uncountable sets. 1+2

(2) Define Cauchy sequence. Show that a Cauchy sequence of real numbers is bounded. 1+3

(3) Prove that a subset A of a metric space X has empty interior if and only if A^c is dense in X. 6

(4) Justify that the set of rational numbers is not complete (in the sense that it does not satisfy the lub axiom). 4

(5) Show that every connected subset of a metric space is contained in a component. 4

(6) Let $\alpha(x) = f(x) = \left\{ \begin{array}{ll} 0, & 0 \leq x < 1 \\ 1, & 1 \leq x \leq 2 \end{array} \right\}$ 3+3

and $g(x) = \left\{ \begin{array}{ll} 0, & 0 \leq x \leq 1 \\ 1, & 1 < x \leq 2 \end{array} \right\}$ Then answer the following:

(i) Is $f \in \mathcal{R}(\alpha)$? If so compute $\int_0^2 f d\alpha$.

(ii) Is $g \in \mathcal{R}(\alpha)$? If so compute $\int_0^2 g d\alpha$.

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