TEZPUR UNIVERSITY
Assignment Spring 2022
MMS102: LINEAR ALGEBRA

Full Marks: 30

The figures in the right-hand margin indicate the marks for the individual questions.

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1. Show that the 4 vectors

$$v_1 = (2,1,3,1),\ v_2 = (-1,0,1,2),\ v_3 = (3,2,7,4),\ v_4 = (1,2,0,-1)$$

   are linearly dependent, and find three of them that are linearly independent. 6

2. Give a basis for the kernel of the linear map $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by 6

$$T(x,y,z) = (x-2y+z, 2x-3y+z, 3x-y-2z).$$

3. Let $V = \mathbb{R}^2$, and define $f_1, f_2 \in V^*$ by $f_1(x,y) = -x+3y$, $f_2(x,y) = x-2y$. Prove that $\{f_1, f_2\}$ is a basis for V^*, and then find a basis for V for which it is the dual basis. 6

4. Show that the matrix $\begin{bmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{bmatrix}$ is not diagonalizable. 6

5. In $\mathbb{R}^4$, let $w_1 = (1,0,1,0), w_2 = (1,1,1,1)$, and $w_3 = (0,1,2,1)$. Find an orthonormal basis for $\operatorname{span}(\{w_1, w_2, w_3\})$. 6

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