Centre for Distance and Online Education
TEZPUR UNIVERSITY
Assignment Spring 2022

Total Marks : 30

1. Let G be an abelian group. Let $x, y \in$ ⌐ ⌐ ⌐ that $o(x) = 5$ and $o(y) = 7$ then find the order of $(xy)^{14}$. **3**

2. Show that any quotient group of an abelian group is abelian. **4**

3. Suppose that H_1 and H_2 are two subgroups of a group G. If $|H_1| = 36$ and $|H_2| = 70$ then find all the possible values of $|H_1 \cap H_2|$. **4**

4. Let G be a group and H a non empty subset of G. Prove that H is a subgroup of G if and only if $ab^{-1} \in H$ for all $a, b \in H$. **4**

5. Let α and β be two elements of S_8 such that
$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 5 & 1 & 6 & 4 & 2 & 7 & 3 \end{pmatrix} \text{ and } \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 4 & 8 & 1 & 7 & 2 & 5 & 3 \end{pmatrix}.$$
Find $\beta\alpha\beta^{-1}\alpha^{-1}$. Also find the order of $\beta\alpha\beta^{-1}\alpha^{-1}$. **3+2=5**

6. Let $f \colon R \to S$ be a ring homomorphism.
   (a) What is $\mathrm{Ker} f$? **1**
   (b) Show that $\mathrm{Ker} f$ is an ideal of R. **2**
   (c) Show that any ideal of R is a kernel of some ring homomorphism. **2**

7. Let A_n and B_n denote the set of even and odd permutations of S_n, for $n \geq 2$, respectively. Show that $|A_n| = |B_n|$. Also find the order of B_n. **5**

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