Department of Mathematical Sciences

Course Structure and Syllabus of Ph.D. Course-work in Mathematical Sciences

Core Course

Course Code	Course Name	L-T-P	CH	Credit
MS 750	Research Methodology in Mathematical Sciences	2-0-2	6	4
RP799	Research and Publication Ethics	1-0-1	2	2

Elective Courses

Course Code	Course Name	L-T-P	CH	CR
MS 753	Magnetohydrodynamics	2-2-0	4	4
MS 754	Programming & Numerical Methods	2-0-2	6	4
MS 755	Ramanujan's Theta Functions and Applications to Number Theory	2-2-0	4	4
MS 756	Topological Structures	2-2-0	4	4
MS 757	Fuzzy Sets & Fuzzy Logic	2-2-0	4	4
MS 758	Algebraic Methods in Operator Theory	2-2-0	4	4
MS 759	Probability Measure, Random Variable and Probability Distribution	2-2-0	4	4
MS 760	Operators on Spaces of Analytic Functions	2-2-0	4	4
MS 761	Theory of Distributions and Sobolev Spaces	2-2-0	4	4
MS 762	Finite Element Methods (FED) for PDEs	2-2-0	4	4
MS 764	Advanced Matrix Theory	2-2-0	4	4
MS 765	Non-Negative matrix Theory	2-2-0	4	4
MS 766	Number Fields and Elliptic Curves	2-2-0	4	4
MS 767	Theory of Rings and Modules	2-2-0	4	4
MS 768	Theory of Groups	2-2-0	4	4
MS 769	Character Theory of Finite Groups	2-2-0	4	4
MS 770	Error Control Codes I	2-2-0	4	4
MS 771	Error Control Codes II	2-2-0	4	4
MS 772	Advanced Fluid Dynamics	3-1-0	4	4
MS 773	Computational Fluid Dynamics	3-1-0	4	4
MS 774	Finite Fields-I	2-2-0	4	4
MS 775	Finite Fields-II	2-2-0	4	4
MS 776 **(AC.41/2022/1/2.2)**	Topics in Nonlinear Analysis and Partial Differential Equations	3-1-0	4	4
MS 777 **(AC.41/2022/1/2.2)**	Topics in Dynamical Systems and Control Theory	3-1-0	4	4

MS778 **(AC.41/2022/1/2.2)**	Advanced Fourier Analysis	3-1-0	4	4
MS 779 **(AC.41/2022/1/2.2)**	Fractional Calculus	3-1-0	4	4
MS 780 **(AC.50/2026/1/5.9)**	Theory of Partitions	2-2-0	4	4
MS 781 **(AC.50/2026/1/5.10)**	General Theory of Relativity and Gravitational Waves	2-2-0	4	4

L: Lectures T: Tutorials P: Practical CH: Contact Hours (all per week) CR: Credit

*A student has to complete total 16 Credit including Research Methodology in Mathematical Sciences (MS 750); any two from the above elective courses**, i.e. Total Credit form elective courses** to be completed by a student = **8,** one CBCT course of CR-4 offered by other departments. Elective courses (as and when offered shall be open as CBCT to other departments.*

Detailed Syllabus

Core Course

MS 750: Research Methodology in Mathematical Sciences (L2-T0-P2-CH6-CR-4)

Unit 1- Introduction
What research basically means, Research need and implication, Brief history and expectation, Research ethics, Plagiarism, Prospects of a researcher career, Basic preparation and requirements.

Unit 2- Objectives and types of research
Motivation and objectives, Research methods vs. methodology, Type of research- Descriptive vs. Analytical, Applied vs. Fundamental, Quantitative vs. Qualitative, Conceptual vs. Empirical.

Unit 3- Research Formulation
Defining and formulating research problem, Selecting problem, Necessity of defining the problem, Importance of literature review in defining a problem, Literature review-primary and secondary sources of literature- reviews, treatise, monographs-patents, web as a source-searching the web, critical literature review- identifying gap areas from literature review, Development of working hypothesis. Formulation and steps of the research proposal.

Unit 4- Research design and method
Basic Principles, Need of research design-Feature of a good design, Important concepts relating to research design- Observations and Facts, Laws and Theories, Prediction and Explanation, Induction, deduction, development of Models. Research plan development-Exploration, Diagnosis, Experimentation. Experiment and Sample designs.

Unit 5- Computer Applications
Utility software- Public license and Proprietary Software, Data analysing software, Typesetting software, Mathematical software. Software for plagiarism.

Unit 6- Data collection and Analysis
Observation and collection of data- primary and secondary data, Methods of collection of data-Sampling methods, Data processing and analysing strategies, Data analysis with statistical methods, Hypothesis testing, Generalization and Interpretations

Unit 7-Reporting and thesis writing
Structure and components of scientific reports- Poster, Journal paper, Technical report and Thesis, Book and book chapters. Significance of different steps in preparation-Layout, Structure and Language of different reports, Illustrations and Tables, Bibliography, Referencing and Foot note, End note. Oral presentation-planning, Preparation, Practice, Making presentation-use of Audio-visual aid, Importance of effective communication.

Unit 8 – Documentation and research output assessment
Research communication and publication, Impact factor and citation, Scientific Citation Index and Extended list, H-index and i-10 index, Patent and royalty.

Unit 9 – Application of result and ethics
Impacts- academic, environmental, industrial, market, social. Ethical issues, ethical committees, commercialization, copyright, royalty, intellectual property rights, reproduction and published materials, plagiarism, reproducibility and accountability of results.

Text Books:

1. C. R. Kothari, *Research Methodology*, 2nd Ed. Wiley Eastern, New Delhi, 1985.
2. Anthony, M., Graziano, A. M. and Raulin, M.L., *Research Methods: A Process ofInquiry*, Allyn and Bacon, 2009.

Reference Book

1. John W Best, V. Kahn, *Research in Education*, 8th Ed. PHI Publication, 1998.
2. K.N. Krishna Swami and others, *Management Research Methodology-Integration of principles, methods and Techniques*, 1st Ed. Pearson Education, 2009.
3. Ranjit Kumar, *Research Methodology-A step by step guide for beginners*, 2nd Ed. Pearson Education, 2005.

RP 799: Research and Publication Ethics (L1-T0-P1-CH2-CR-2)

Unit 1- Philosophy and Ethics

1. Introduction to philosophy: definition, nature and scope, concept, branches
2. Ethics: definition, moral philosophy, nature of moral judgements and reactions

Unit 2- Scientific Conduct

1. Ethics with respect to science and research
2. Intellectual honesty and research integrity
3. Scientific misconducts: Falsification, Fabrication, and Plagiarism (FFP)
4. Redundant publications: duplicate and overlapping publications, salami slicing
5. Selective reporting and misrepresentation of data

Unit 3- Publication Ethics

1. Publication ethics: definition, introduction and importance
2. Best practices / standards setting initiatives and guidelines: COPE, WAME, etc.
3. Conflicts of interest
4. Publication misconduct: definition, concept, problems that lead to unethical behavior and vice versa, types
5. Violation of publication ethics, authorship and contributorship
6. Identification of publication misconduct, complaints and appeals
7. Predatory publishers and journals

Practice

Unit 4- Open Access Publishing

1. Open access publications and initiatives
2. SHERPA/RoME0 online resource to check publisher copyright & self-archiving policies
3. Software tool to identify predatory publications developed by SPPU
4. Journal finder / journal suggestion tools viz. JANE, Elsevier Journal Finder, Springer Journal Suggester, etc.

Unit 5- Publication Misconduct

A. Group Discussions (2 hrs.)

1. Subject specific ethical issues, FFP, authorship
2. Conflicts of interest
3. Complaints and appeals: examples and fraud from India and abroad

B. Software tools (2 hrs.)

Use of plagiarism software like Turnitin, Urkund and other open source software tools

Unit 6- Databases and Research Metrics

A. Databases (4 hrs.)

1. Indexing databases
2. Citation databases: Web of Science, Scopus, etc.

B. Research Metrics (3 hrs.)

1. Impact Factor of journal as per Journal Citation Report, SNIP, SJR, IPP, Cite Score
2. Metrics: h-index, g index, il0 index, altmetrics

Reference Book

1. A. Bird, *Philosophy of Science*, (Routledge, 2006).
2. A. MacIrityre, *A Short History of Ethics*, (London, 1967).
3. P. Chaddah, *Ethics in Competitive Research: Do not get scooped; do not get plagiarized*, (2018.ISBN:978-9387480865).
4. National Academy of Sciences, National Academy of Engineering and Institute of Medicine. *On Being a Scientist: A Guide to Responsible Conduct in Research,* (Third Edition. National Academies Press., 2009).
5. D. B. Resnik, *What is ethics in research & why is it important.* (National Institute of

Environmental Health Sciences, 1-10, 2011. Retrieved from https://www.niehs.nih.gov/research/resources/bioethics/whatis/index.cfm)
6. J. Beall, *Predatory publishers are corrupting open access*. (Nature, 489(7415), 2012, 179-179. https://doi.org/10.1038/489179a).
7. Indian National Science Academy (INSA), *Ethics in Science Education, Research and Governance*, (2019, ISBN:978-81-939482-1-7. http://www.insaindiaxes.in/pdf/Ethics Book.pdf)

Elective Courses

(Any two are to be chosen by a student)

MS 753 Magnetohydrodynamics (L2-T2-P0-CH4-CR4)

Unit-1
The MHD approximations, The Kinematic aspects of MHD.

Unit-2
The magnetic force and its effects.

Unit -3
Linear MHD, Non-dimensional numbers in MHD,

Unit-4
Alfrin's theorem and some consequences of Alfrin's theorem.

Unit-5
Stability of MHD Flows, turbulence in MHD.

Textbook(s)

1. Shercliff, J. A. *A Text Book of Magnetohydrodynamics* (Pergamon, New York, 1965).

Reference book(s)

1. Ferraro, V. C. A. & Plumpton, C. *An Introduction to Magneto-fluid Mechanics* (Oxford University Press, 1961).
2. Pai, S. I. *Magnetohydrodynamics and Magnetogasdynamics* (PN, 1955).

MS 754 Programming and Numerical Methods (L2-T0-P2-CH6-CR4)

Unit-1
Approximation and algorithms.

Unit-2
Interpolation: Lagrange interpolation, finite differences, iterated and inverse interpolation, Hermite interpolation.

Unit-3
Numerical integration: Newton-Cotes formulas, Gaussian quadrature, composite quadrature formulas, adaptive integration.

Unit-4
Functional approximation, least squares approximation, minimum-maximum error techniques. Chebyshev polynomials.

Unit-5
Solution of non-linear equations: functional iteration, bisection, secant, Newton-Raphson. Solving problems with FORTRAN 77.

Unit-6
Solution of linear systems: direct methods- Gauss elimination, LU decomposition matrix inversion. Solving problems with FORTRAN 77.

Unit-7
Iterative methods: Jacobi Method, Gauss-Seidel method, SOR method. Eigenvalue problems: power, inverse power method. Solving problems with FORTRAN 77.

Unit-8
Numerical solutions of ODE's: Taylor series, Euler and Runge-Kutta methods. Solving problems with FORTRAN 77.

Textbook(s)

1. F. B. Hilderbrand, *Introduction to Numerical Analysis* (Tata McGraw Hill, New Delhi, 1974).
2. S. S. Sastry, *Introductory methods of Numerical Analysis* (Prentice Hall of India, New Delhi, 1977).

Reference book(s)

1. M. K. Jain, S. R. K. Iyengar, R. K. Jain, *Numerical methods, Problems and solutions*, (New Age International (P) Ltd., 1996).

MS 755 Ramanujan's Theta Functions & Applications to Number Theory

(L2-T2-P0-CH4-CR4)

Unit-1

Ramanujan's general theta-function, special cases and their relations, *q*-series and infinite products, Jacobi triple product identity.

Unit-2

Schröter's formulae and theta-function identities and Ramanujan's Modular equations.

Unit-3

Class invariants, Evaluation of class invariants, Explicit values of theta-functions

Unit-4

Ramanujan's continued fractions and explicit values.

Unit-5

Applications of theta-functions and modular equations to the theory of partitions.

Unit-6

Ramanujan's famous congruences for the partition function, Rogers-Ramanujan-type functions and partition theoretic interpretations.

Textbook(s)

1. Berndt, B. C. *Number Theory in the Spirit of Ramanujan* (AMS, 2006).
2. Andrews, George E., *The Theory of Partitions (A*ddition-Wesley, Reading, MA, 1976).

Reference book(s)

1. Berndt, Bruce C., *Ramanujan's Notebooks*, Part III, IV and V (Springer, 1991, 1994, 1998).
2. Whittaker, E. T. and Wilson, G. N., *A Course in Modern Analysis (*Cambridge University Press, Cambridge, 1966. Indian edition is published by Universal Book Stall, New Delhi, 1991).
3. Agarwal, R. P**.**, *Resonance of Ramanujan's Mathematics*, Vol. I & II (New Age International (P) Limited, New Delhi, 1996).
4. Hardy, G. H., *Ramanujan (*AMS-Chelsea, New York 1999).

MS 756 Topological Structures **(L2-T2-P0-CH4-CR4)**

Unit-1

Basics of point set topology

Unit-2

Uniform structures, uniform continuity, completeness and completions, metrizability of uniform spaces; different uniform structures in topology,

Unit-3

Paracompactness, countably paracompact space, weakly and strongly paracompact space, metrization results;

Unit-4

Dimension theory, basic properties of three dimension functions ind, Ind and dim, properties of dim, the imbedding theorem.

Textbook(s)

1. Willard, S., *Topology* (Addision -Wesley, Reading, 1970)
2. Kelley, J.L.,*Topology (*Graduate texts in Mathematics, Vol. 27, Springer, 1991).

Reference Book(s)

1. Bourbaki,N., *Elements of Mathematics: General Topology*, Vols I & II (Springer-Verlag, 1988).
2. Engelking, R., *Topology (*Heldermann Verlag, 1989).
3. Munkers, J.R., Topology (Pearson Rducation Inc., 2000)

MS 757 Fuzzy sets and Fuzzy Logic (L2-T2-P0-CH4-CR4)

Unit-1
Fuzzy sets - basic definitions, alpha-level sets, convex fuzzy sets, basic operations on fuzzy sets, types of fuzzy sets, cartesian products, algebraic products, bounded sum and difference, t-norms and t-conorms.

Unit-2
The extension principle - the Zadeh's extension principle, image and inverse image of fuzzy sets, fuzzy numbers, elements of fuzzy arithmetic.

Unit-3
Fuzzy relations and fuzzy graphs, composition of fuzzy relations, min-max composition and its properties, fuzzy equivalence relations, fuzzy graphs.

Unit-4
Fuzzy logic, fuzzy propositions, fuzzy quantifiers, linguistic variables, inference from conditional fuzzy propositions, compositional rule of inference.

Unit-5
Approximate reasoning - an overview of fuzzy expert systems, fuzzy implications and their selection, multi-conditional approximate reasoning, role of fuzzy relation equation. Rough sets, intuitionistic fuzzy sets and applications

Textbook(s)

1. G. J. Klir and B. Yuan, *Fuzzy Sets and Fuzzy Logic: Theory and Applications (*Prentice Hall of India, New Delhi, 1997).
2. H. J. Zimmermann, *Fuzzy set theory and its Applications (*Allied publishers Ltd., New Delhi, 1991).

Reference book(s)

1. D. Dubois and H. Prade, *Fuzzy sets and systems: theory and applications (*Academic Press, New York, 1980).
2. A. Kandel, F*uzzy mathematical techniques with applications (*Addison-Wesley, Reading, Mass, 1986).
3. A. Kaufmann, and M. M. Gupta, *Introduction to fuzzy arithmetic: theory and applications* (Van Nostrand Reinhold, New York, 1985).
4. B. Kosko, *Fuzzy thinking: the new science of fuzzy logic* (Flamingo, 1994).

MS 758 Algebraic Methods in Operator Theory (L2-T2-P0-CH4-CR4)

Unit-1
Fredholm operators, Semi-Fredholm operators, index of a Fredholm (semi-Fredholm) operator.

Unit-2
Weighted shifts and their norm and spectral radii.

Unit-3
Normaloid, convexoid and spectraloid operators **.**

Unit-4
Toeplitz operators **.**

Unit-5
Transitive, Reductive,Reflexive operator algebras.

Unit-6
Strictly cyclic and cyclic operator algebras, triangular and quasi triangular operator algebras.

Unit-7
The Hardy Spaces, Beurling's Theorem and its extensions, The Inner-Outer Factorization of Functions in H^2.

Textbook(s)

1. R. G. Douglas, *Banach Algebra Techniques in Operator Theory* (Academic Press 1972).
2. Conway, J. B., *A course in Operator Theory* (AMS., GSM Vol. 21, 1999).
3. Abramovich, Y. A. and Aliprantis, C. D., *An Invitation to Operator Theory* (AMS, GSM Vol. 50, 2002).

Reference book(s)

1. H. Radjavi and P. Rosenthal**,** *Invariant Subspaces*, Springer Verlag, 1973.
2. D. A. Herrero**,** *Approximation of Hilbert Space Operators I*, PitmanAdvanced Publishing Program, 1982.
3. D. A. Herrero, *Approximation of Hilbert Space Operators II*, PitmanAdvanced Publishing Program, 1984.
4. P. R. Halmos, *A Hilbert Space Problem Book*, III Edn. D. Van Nostrand Co., 1974.
5. Bernard Beauzamy, *Introduction to Operator Theory and Invariant Subspaces*, North Holland, 1987.
6. M. Rosenblum and J. Rovnyak, *Hardy Classes and Operator Theory*, Dover Publications, Inc., 1997.

MS 759 Probability Measure, Random Variable and Probability Distribution

(L2-T2-P0-CH4-CR4)

Unit-1

Definition of probability measure on a sample space, Lebesgue measure, measurable sets, measurable functions and random variables. Probability distribution of a random variable.

Unit-2

Lebesgue Integral and Expectation of a random variable and Extension theorem.

Unit-3

Probability measures defined on complete, separable metric spaces. The concept of tightness, Weak convergence, Portmanteau theorem.

Unit-4

Prohorov's theorem. Weiner Measure, Brownian Bridge.

Unit-5

Sequences of random variables, almost sure convergence, convergence in probability and convergence in law of distributions. Laws of Large numbers, Central limit theorems, Berry-Esseen theorem. Empirical process and Glivenko-Cantelli theorem.

Textbook(s)

1. Billingsley, P**.** *Convergence in Probability Measures*, (John Wiley and Sons, 1999).
2. Chow and Teicher, *Probability Theory*, *Independence, Interexchangeabilities*, *Martingales*, (Springer-Verlag, 1997).

Reference book(s)

1. Ash, R. B. *Probability and Measure Theory*, *Second Edition* (Harcourt/Academic Press, 2000).
2. Feller, W. *An Introduction to Probability Theory and Its Applications,* Vol. II (John Wiley and Sons, 1965).

MS 760: Operators on Spaces of Analytic Functions (L2-T2-P0-CH4-CR4)

Unit-1

Preliminaries in Operator Theory : Operators on Banach Spaces and Hilbert Spaces, Compact operators, Schatten class operators, Hilbert-Schmidt operators.

Unit-2

Theory of Bergman Spaces : Bergman type projections, Bergman metric, Atomic decomposition, The Bloch space.

Unit-3

A study of Hardy spaces H^p with special reference to the Hardy-Hilbert space H^2, Hankel and Toeplitz operators on the H^2 space.

Unit-4

Hankel and Toeplitz operators on the Bergman space, Composition operators on the Hardy and Bergman spaces.

Textbook(s)

1. Ruben A. Martinez-Avendano and Peter Rosenthal, *An introduction to Operators on the Hardy-Hilbert Space* (Springer, 2007).
2. Kehe Zhu, *Operator Theory in Function Spaces*, Second Edition (Marcel Dekker, 2007).
3. John B. Conway, *A course in Operator Theory* (American Mathematical Society, 1999).

Reference book(s)

1. Peter L. Duren, Alexander Schuster, *Bergman Spaces* (American Mathematical Society, 2004).
2. Haakan Hedenmalm, Boris Korenblum, Kehe Zhu, *Theory of Bergman Spaces* (Springer, 2000).
3. Ronald G. Douglas, *Banach Algebra Techniques in Operator Theory*, Second Edition, (Springer, 1998).
4. John B. Conway, *A course in Functional Analysis* (Springer-Verlag, 1997).
5. Kenneth Hoffman, *Banach Spaces of Analytic Functions* (Dover, 1988).

MS 761 Theory of Distribution and Sobolev Spaces (L2-T2-P0-CH4-CR4)

Unit -1

Inner Measure, Outer Measure, Measurable Sets, Lebesgue Measure, Measurable functions, Lebesgue Integration, L_p spaces.

Unit -2

Test functions and distribution, Generalised derivatives, Some operations on distribution, The Fourier transform, The inverse Fourier transform.

Unit -3

Sobolev norms and associated spaces, Extension theorems, Inclusion relations and Sbolev's inequality, Negative norms and Duality, Fractional order Sobolev space, trace Theorems

Unit -4

Interpolation, Bounds for the interpolation error, Inverse estimates, Interpolation of non-smooth functions

Textbook(s)

1. B. Daya Reddy, *Introductory Functional Analysis : With Applications to Boundary Value Problems and Finite Elements* (Springer, 1997).
2. Robert S. Strichartz , *A guide to Distribution Theory and Fourier Transforms,* Studies in Advanced Mathematics (CRC Press, USA ,1994).
3. S. Kesavan, *Topics in Functional Analysis and Applications (*Wiley Eastern Limited, 1989).

Reference book(s)

1. R. A. Adams and John J. F. Fournier, *Sobolev Spaces*, (Academic Press, 2003).
2. Soo-Bong Chae, *Lebesgue Integration*, (Springer-Verlag, 1994).
3. Elliot H. Lieb and Michel Loss, *Analysis (*Narosa Publishing House, New Delhi, 1997).

MS 762 Finite Element Methods (FEM) for PDEs (L2-T2-P0-CH4-CR4)

Unit -1

Variational formulation**,** Lax-Milgram Theorem**,** Regularity estimates**,** Construction of Finite Element Space, Finite Element Approximation to Elliptic BVP, Convergence Analysis

Unit -2

Variational formulation, Semi-discretization, Discretization in space and time, The backward Euler and Crank Nicolson methods, Error Analysis

Unit-3

Variational formulation, Semi-discretization, Fully discretization, Error Analysis.

Effect of Numerical Quadrature: Curved Elements, Numerical quadrature, Isoparametric Finite Elements.

Unit -4

Introduction to Mixed FEM, Discontinuous Galerkin Method

Textbook(s)

1. S.C. Brenner and L.R. Scott, The Mathematical Theory of Finite Element Methods, (Springer-Verlag, 1994).
2. J. N. Reddy, An Introduction to the Finite Element Method (McGraw-Hill, Inc., 1993).
3. S. Larsson and V. Thomée, Partial Differential Equations with Numerical Methods, (Springer, 2005).

Reference book(s)

1. P. G. Ciarlet, The Finite Element Method for Elliptic Problems (North Holland, Amsterdam, 2002)
2. V. Thomee, Galerkin Finite Element Method for Parabolic Problems, (Springer- Verlag, 1997).
3. C. Johnson, Numerical Solution of Partial Differential Equations by Finite Element Method (Dover Publications, 2008).

MS 764 Advanced Matrix Theory (L2-T2-P0-CH4-CR4)

Prerequisites: MS 403 Linear Algebra

Unit-1
Review of Linear Algebra

Unit-2
Unitary matrices, Unitary equivalence, Normal matrices, Schur's theorem.

Unit-3
Hermitian and symmetric matrices, variational characterization of eigenvalues , application of variational characterizations.

Unit-4
Positive definite matrices, Positive semidefinite matrices.

Unit-5
Singular value decomposition, Schur complement.

Textbook(s)

1. Horn, R. and Johnson, C. R., *Matrix Analysis (*Cambridge University Press, Cambridge, 1985).
2. Meyer, Carl D. *Matrix Analysis and Applied Linear Algebra (*SIAM, 2000).

Reference book(s)

1. Bellman, R., *Introduction to Matrix Analysis (*McGRAW-HILL BOOK COMPANY, 1960).
2. Horn, R. A. and Johnson C. R., *Topics in Matrix Analysis* (Cambridge University Press, Cambridge, 1999 (Reprint)).

MS 765 Non-Negative Matrix Theory ***(L2-T2-P0-CH4-CR4)***

Prerequisites: MS 403 Linear Algebra

Unit-1
Nonnegative matrices, positive matrices, Irreducible matrices, Reducible matrices.

Unit-2
Perron's theorem, Primitive matrices.

Unit-3
Stochastic matrices, Graphs and nonnegative matrices.

Unit-4
Inverse eigenvalue problem.

Textbook(s)

1. Minc, Henryk, *Nonnegative Matrices* (John Wiley and Sons, 1988).
2. Horn, R. and Johnson, C. R., *Matrix analysis* (Cambridge University Press, Cambridge, 1985).

Reference book(s)

1. Berman and Plemmons, *Nonnegative Matrices in the Mathematical Sciences*, (SIAM, 1994).
2. Bapat and Raghavan, *Nonnegative Matrices and Applications* (CUP, 1997).

MS 766 Number Fields and Elliptic Curves (L2-T2-P0-CH4-CR4)

Unit-1
Number fields and their rings of integers, Prime decomposition in number rings.

Unit-2
The ideal class group and the unit group, Dirichlet's Unit Theorem, Dedekind zeta function and the class number formula, Class Numbers of Quadratic Fields and Cyclotomic fields.

Unit-3
Introduction to algebraic curves, singular and non-singular curves, Mordell-Weil group law on elliptic curve, explicit formulas for group law.

Unit-4
Points of finite order on elliptic curves and Nagell-Lutz theorem.

Unit-5
Mordell's theorem, rank of elliptic curves.

Unit-6
Elliptic curves over finite fields, Complex multiplication.

Textbook(s)

1. Marcus, D. A. *Number Fields*, 3th edition (Springer-Verlag, 2009).
2. Stewart, I. N. & Tall, D. *Algebraic Number Theory and Fermat's Last Theorem*, 3rd edition (A K Peters Ltd., 2000).
3. Silverman, J. H. & Tate, J. *Rational Points On Elliptic Curves* (Springer- Verlag, 2005).

Reference book(s)

1. Mollin, R. A. *Algebraic Number Theory* (CRC Press, 1999).
2. Esmonde, J & Murty, R. M. *Problems in Algebraic Number Theory* (GTM Vol. 190, Springer-Verlag, 2000).
3. Knapp, A. W. *Elliptic Curves* (Princeton University Press, 1993).

MS 767 Theory of Rings and Modules (L2-T2-P0-CH4-CR4)

Unit-1

Idempotent, Nilpotent and Von Neumann regular elements, Chain conditions.

Unit-2
Artinian & Noetherian rings and modules, Exact sequences

Unit-3
Essential (Large) and Superfluous (Small) submodules, Relative complement, Closed submodule, Semisimple rings and modules

Unit-4
Jacobson radical, Jacobson semisimple (semiprimitive) rings, Local, Semilocal rings.

Unit-5
Decomposition theorems (Fittings', Peirce etc.), Corner rings, Uniform submodules and finite dimensionality.

Unit-6
Projective and Injective submodules.

Textbook(s)

1. Lam, T. Y. *A First course in Noncommutative rings*, 2nd edition (Springer, 2001).
2. Goodearl, K. R. *Ring Theory (*Marcel Dekker, Inc. 1976).
3. Anderson, F. W. and Fuller, K. R. *Rings and Categories of Modules,* 2nd edition (Springer-Verlag, 1992).

Reference book(s)

1. Musili, C. *Introduction to Rings and Modules,* 2nd revised edition (Narosa Publishing House, 1994).
2. Lambek, J. *Lectures on Rings and Modules,* 3rd edition *(*AMS Chelsea Publishing, 2009).
3. Burton, D.M. *A first course in rings and ideals* (Addison-Wesley, 1970).
4. Barshay, J. *Topics in ring theory* (W.A. Benjamin Publ. 1969).
5. Chatters, A.W. and Hajarnavis, C.R. *Rings with chain condition* (Pitman, 1980).

MS 768 Theory of Groups **(L2-T2-P0-CH4-CR4)**

Unit -1

Group action, orbit and stabilizer, applications of group action, Sylow theorems and application, transitive and primitive action, Frattini argument, Frobenius group and Frobenius theorem, complement of normal subgroup, Schur-Zassenhaus theorem.

Unit -2

Commutator identities, commutator subgroups, derived series, lower and upper central series, Nilpotent group, The Frattini subgroup, p-groups, extra-special p-groups, Solvable and supersolvable groups, symmetric groups and alternating groups.

Unit - 3

Free groups, presentations of groups, presentation of symmetric group.

Unit - 4

Extension of a group, short exact sequence, automorphism group, complete group, semidirect product, wreath product, central product of groups, transfer homomorphism and its applications.

Textbook(s)

1. Rotman, J. J., *An Introduction to the Theory of Groups*, Fourth Edition (Springer 1995).
2. Kurzweil, H. and Stellmacher B., *The theory of finite groups*, (Springer 2004).

Reference Book(s)

1. Robinson, D. J. S., *A Course in the Theory of Groups*, Second Edition (Springer 1996).
2. Isaacs, I. M., *Finite Group Theory*, (American Mathematical Soc., 2008).
3. Aschbacher, M., *Finite Group Theory*, (Cambridge University Press, 2000)

MS 769 Character Theory of Finite Groups (L2-T2-P0-CH4-CR4)

Unit -1

Algebras and modules, group algebra, centre of a group algebra, class sums, homomorphisms, irreducible and indecomposable modules, completely reducible modules, semisimple algebra, Mashke's theorem, Wedderburn theorem.

Unit -2

Representation of an algebra, representation of a finite group, similar and irreducible representations, characters afforded by a representation, linear and non-linear characters, regular character, irreducible characters .

Unit -3

Class functions, Schur's lemma, orthogonality relations, kernel of a character, restriction of characters to a subgroup, construction of character table for some groups of small order, interpretation of character table of a group.

Unit -4

Algebraic integers, character values as algebraic integers, Burnside theorem for irreducible characters, product of characters, induced character, Frobenius reciprocity.

Textbook(s)

1. Isaacs, I. M., *Character Theory of Finite Groups*, (Dover Publications, Inc., New York, 1994).

Reference Book(s)

1. Huppert, B., *Character Theory of Finite Groups*, (Walter de Gruyter, New York, 1998).
2. Berkovich, Y. G. and Zhmud, E. M., *Characters of Finite Groups Part 1*, (American Mathematical Soc., 1997).

MS 770 Error Control Codes I (L2-T2-P0-CH4-CR4)

Unit-1

Linear codes, Generator and parity check matrices, Counting of generator matrices, Weights and distances, Permutation equivalent codes, More general equivalence of codes, Encoding, decoding, and Shannon's Theorem.

Unit-2

Bounds on the size of codes: $A_q(n, d)$ and $B_q(n, d)$, The Plotkin Upper Bound, The Johnson Upper Bounds, The Singleton Upper Bound and MDS codes, The Elias Upper Bound, Asymptotic bounds, Lexicodes.

Unit-3

Cyclic codes: Factoring $x^n - 1$, Basic theory of cyclic codes, Idempotents and multipliers, Zeros of a cyclic code, Minimum distance of cyclic codes, Meggitt decoding of cyclic codes, Affine-invariant codes, Hamming and Golay codes as Cyclic codes.

Unit-4

Weight distributions: The MacWilliams equations, Equivalent formulations, A uniqueness result, Weight distribution of MDS codes, Coset weight distributions.

Textbook(s)

1. W. C. Huffman and V. Pless, *Fundamentals of Error-Correcting Codes*, Cambridge UniversityPress, Cambridge, Reprint, 2010.
2. S. Ling and C. Xing, *Coding Theory-A First Course*, Cambridge University Press, 2004.

Reference Book(s)

1. W.W. Peterson and E.J. Weldon, Jr., *Error-Correcting Codes*, M.I.T. Press, Cambridge, Massachusetts, 1972.
2. S. Lin and D. J. Costello, *Error Control Coding-Fundamentals and Applications*, Pearson Education India, 2011.
3. J.H. Van Lint, *Introduction to Coding Theory*, Graduate Texts in Mathematics, 86, Springer, 1998.
4. F. J. MacWilliams and N. J. A. Sloane, *The Theory of Error-Correcting Codes*, North -Holland, 1983.

MS 771 Error Control Codes II **(L2-T2-P0-CH4-CR4)**

(Pre-requisite MS 770 Error Control Codes I)

Unit-1

Self-dual codes: The Gleason–Pierce–Ward Theorem, Gleason polynomials, Upper bounds, The Balance Principle and the shadow, Counting self-orthogonal codes.

Unit-2

Covering radius and cosets: Basics, The Norse Bound and Reed–Muller codes, Covering radius of BCH codes, Covering radius of self-dual codes.

Unit -3

Codes over Z_4: Basic theory of Z_4 -linear codes, Binary codes from Z_4 -linear codes, Cyclic codes over Z_4 , Self-dual codes over Z_4 , Kerdock codes, Preparata codes.

Unit -4

Convolutional codes : Burst-Error-Correcting Convolutional Codes, Bounds on Burst Error -Correcting Capability, Interleaved Cyclic and Convolutional Codes.

Textbook(s)

1. W. C. Huffman and V. Pless, *Fundamentals of Error-Correcting Codes*, Cambridge University Press, Cambridge, Reprint, 2010.
2. S. Lin and D. J. Costello, *Error Control Coding-Fundamentals and Applications*, Pearson Education India, 2011.

Reference Book(s)

3. S. Ling and C. Xing, *Coding Theory-A First Course*, Cambridge University Press, 2004.
4. W.W. Peterson and E. J. Weldon, Jr., *Error-Correcting Codes*, M.I.T. Press, Cambridge, Massachusetts, 1972.
5. Zhexian Wan, *Quaternary Codes*, World scientific publishing Co. Pte. Ltd., 1997.
6. J.H. Van Lint, *Introduction to Coding Theory*, Graduate Texts in Mathematics, 86, Springer, 1998.

MS 772 Advanced Fluid Dynamics (L3-T1-P0-CH4-CR4)

Unit-1
Exact solutions of incompressible Navier-Stokes equations: Plane Poiseuille flow and Couette flow, Hagen-Poiseuille flow, flow between two concentric rotating cylinders, Stokes first and second problems.

Unit-2
Slow viscous flow: Stokes and Oseen's approximation.

Unit-3
Boundary layer Analysis: Derivation of boundary layer equations; Exact solutions; Approximate methods; Momentum integral method.

Unit-4
Introduction to Turbulence: Description of turbulent flow, averaging, RANS, Introduction to turbulent models, Empirical laws.

Unit-5
Introduction to Compressible flow: Review of conservation laws for compressible flows, speed of sound, wave equation, regimes of Mach number, shocks, wave propagation, sound speed, Mach number, isentropic flow, static and stagnation properties.

Unit-6
One Dimensional Flow: Converging-diverging nozzles, shock waves, moving and reflected waves, blast waves, wind tunnels.

Unit-7
Two Dimensional Flow: Oblique shock wave theory, conical oblique shock waves, Prandtl-Mayer expansion Fans

Textbook(s)

1. Schlichting, H. and Gersten, K. *Boundary Layer theory*, (Springer, 2000).
2. White, F.M. *Viscous Fluid Flow*, (Tata McGraw-Hill, 2011).
3. Anderson, J.D. *Modern Compressible Flow (With Historical Perspective)*, (McGraw-Hill, 1990).

Reference book(s)

1. Yahya, S.M. *Fundamentals of Compressible Flow*, (New Age International, 2010).
2. Kundu P.K. and Cohen I.M. *Fluid Mechanics*, (Elsevier, 2005).

Note:
Prerequisites: *Basic knowledge in Fluid Mechanics.*
Students intended for: *Ph.D.*
Nature: *Elective*
Course Objective: *The course builds on previous concepts learned in basic courses in fluid mechanics. One aim is also to provide students understanding in compressible flow.*

MS 773 Computational Fluid Dynamics (L3-T1-P0-CH4-CR4)

Unit-1
Basics of Finite Difference Methods (FDM): Finite differences, difference representation of PDE, examples, stability considerations.

Unit-2
Applications of FDM to selected model equations: Heat equation, Wave equation, Laplace equation, inviscid Burger equation, viscous Burger equation.

Unit-3
Applications of FDM to the equations of fluid mechanics: Steady convection-diffusion equations, Unsteady convection-diffusion equations. Conservation laws, Reynolds equation, Euler equations, Incompressible plane flows, Stream function and vorticity equations, Method for solving vorticity transport equation,

Unit-4
Finite volume methods: Application to Euler equations, Upwind difference scheme, Viscous flow solutions, Total variation diminishing schemes, Godunov-type schemes.

Unit-5
Simple CFD Techniques: Lax-Wendroff Technique, Mac Cormack's Techniques, Staggered grid, SIMPLE Algorithm. Numerical Solutions of Navier-Stokes equations on collocated and on staggered grids.

Unit-6
Simulation of fluid flow and compact discretization: Flow in a lid driven cavity, Numerical grid generation, Solution of the Navier-Stokes equations in the transformed plane. Higher order compact schemes to the Navier-Stokes equations. Compact schemes for Conservation Laws.

Textbook(s)

1. Anderson, D.A., Tannehill, J.C. and Pletcher, R.H. *Computational Fluid Dynamics and Heat Transfer,* (CRC Press, 2012).
2. Chung, T.J. *Computational Fluid Dynamics,* (Cambridge University Press, 2005).
3. Hoffman, J.D. *Numerical Methods for Engineers and Scientists,* CRC Press, 2001).

Reference book(s)

1. Fletcher, C. A. J. *Computational Techniques for Fluid Dynamics, Volume 1 & 2,* (Springer Verlag, 1992).
2. Anderson, J. D. *Computational Fluid Dynamics – The Basics with Applications* (Mc-Graw Hill, 1995).
3. Ferziger, J.H. and Peric, M. *Computational Methods for Fluid Dynamics* (Springer, 1999).

Note:
Prerequisites: *Basic knowledge in Fluid Mechanics and Partial Differential Equations.*
Students intended for: *Ph.D.*
Nature: *Elective*
Course Objective: *This course will emphasize on the development of basic as well as advance Finite Difference approaches to provide numerical solutions of Partial Differential Equations, frequently arise in the field of Fluid Mechanics. This course is a theoretical and the main objective is to provide basic concepts of Computational Fluid Dynamics, in terms of comprehensive theoretical study and some of its computational aspects.*

MS 774 Finite Fields -I **(L2-T2-P0-CH4-CR4)**

Unit-1

Algebraic Foundations (Recap): Fields, Polynomials and Field extensions.

Unit-2

Structure of Finite Fields: Characterization of Finite Fields, Roots of Irreducible Polynomials, Traces, Norms, and Bases, Roots of Unity and Cyclotomic Polynomials, Representation of Elements of Finite Fields.

Unit-3

Polynomials over Finite Fields: Order of Polynomials and Primitive Polynomials, Irreducible Polynomials, Construction of Irreducible Polynomials, Linearized Polynomials, Binomials and Trinomials.

Unit-4

Factorization of Polynomials: Factorization over Small Finite Fields, Factorization over Large Finite Fields, Calculation of Roots of Polynomials.

Textbook(s)

1. Rudolf Lidl and Harald Niederreiter, *Introduction to finite fields and their applications,* (Cambridge University Press, 2nd Edition, 2012).

Reference book(s)

1. Gary L. Mullen and Daniel Panario, *Handbook of Finite Fields*, (Chapman and Hall/CRC, 1st Edition, 2013).

MS 775 Finite Fields -II **(L2-T2-P0-CH4-CR4)**

Unit-1

Exponential Sums: Characters, Gaussian Sums, Jacobi Sums, Kloosterman Sums.

Unit-2

Linear Recurring Sequences: Feedback Shift Registers, Periodicity Properties, Impulse Response Sequences, Characteristic Polynomial, Generating Functions, he Minimal Polynomial, Families of Linear Recurring Sequences, Characterization of Linear Recurring Sequences, Distribution Properties of Linear Recurring Sequences.

Unit-3

Theoretical Applications of Finite Fields: Finite Geometries, Combinatorics, Linear Modular Systems, Pseudorandom Sequences.

Unit-4

Applications of Finite Fields in Coding Theory, Cryptology and Number Theory: Linear Codes, Cyclic Codes, Goppa Codes. Cryptosystems, Stream Ciphers, Discrete Logarithms, Further Cryptosystems. Elliptic curves over finite fields.

Textbook(s)

1. Rudolf Lidl and Harald Niederreiter, *Introduction to finite fields and their applications,* (Cambridge University Press, 2nd Edition, 2012).

Reference book(s)

1. Gary L. Mullen and Daniel Panario, *Handbook of Finite Fields*, (Chapman and Hall/CRC, 1st Edition, 2013).
2. Lawrence C. Washington, Elliptic Curves, *Number Theory and Cryptography*, (Second Edition, CRC Press, 2008).

MS 776 Topics in Nonlinear Analysis and Partial Differential Equations

(L3-T1-P0-CH4-CR4)

Unit 1

Review of Basics of Lebesgue theory, Distributions, Sobolev Spaces.

Unit 2

Review of methods for linear partial differential equations

Unit 3

Banach space valued differentiation and Bochner integration

Unit 4

Introduction to semigroup of operators, Abstract Cauchy Problem.

Unit 5

C_0 semigroups and their properties

Unit 6

Yosida Approximations, Hille-Yosida theorem and Hille-Yosida-Phillip theorem

Unit 7

Semigroups of Compact operators, Differentiability of semigroups, Analytic semigroups

Unit 8

Abstract Cauchy Problem and Semigroup solutions to homogeneous and Nonhomogeneous initial value problems, Regularity, and asymptotic behaviour of solutions

Textbook:

1. Pazy, A. *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer Verlag, NewYork, AMS, 1983

References:

1. Goldstein, J. A. *Semigroups of Linear Operators and Applications*, Second Edition, Dover Publications, NewYork, 2017
2. Renerdy, M. and Roger, R. C. *An Introduction to Partial Differential Equations*, Springer Verlag, NewYork, 2006.
3. Evan, L. C. *Partial Differential Equations*, AMS, 1998.

MS 777 Topics in Dynamical Systems and Control Theory (L3-T1-P0-CH4-CR4)

Unit 1

Formulation of Physical System, Existence of solutions, Uniqueness of Solutions, The Method of Successive Approximations

Unit 2

Linear Systems: Uncoupled linear Systems, Diagonalization, Exponentials of Operators, The Fundamental Theorem for Linear Systems, Linear Systems in R^2, Complex Eigenvalues, Multiple Eigenvalues.

Unit 3

Jordan Forms, Stability theory, Nonhomogeneous Linear Systems, Linear systems with constant Coefficients, Linear systems with periodic Coefficients.

Unit 4

Nonlinear Systems: The Fundamental Existence-Uniqueness Theorem, Dependence on Initial Conditions and Parameters, The Maximal Interval of Existence, The Flow defined by a differential Equation, Linearization, The Stable Manifold Theorem, The Hartman—Grobman Theorem, Stability, Saddles, Nodes, Foci and Centers.

Unit 5

Definition of controllability and Observability, Kalmann theorem for autonomous systems, Kalman Theorem for non-autonomous system.

Unit 6

Duality Theorem, Optimal control, Companion form, Feedback control.

Unit 7

Stabilization of linear control systems, Sterilizability using Lyapunov theory.

Unit 8

Optimal control of linear system Discrete control system.

Textbook:

1. Curtain, R. F. and Zwart, H. J. *An introduction to infinite-dimensional linear systems theory*, Texts in Applied Mathematics, 21. Springer-Verlag, New York, 1995.

References:

1. Sontag, E. D. *Mathematical Control Theory*, Springer-Verlag, New York 1990.
2. Coddington, E. A. and Levinson, N. *Theory of Ordinary Differential Equations*, Tata Mcgraw Hill, 2012
3. Perko, L. *Differential Equations and Dynamical Systems*, Springer-Verlag, New-York 2001.
4. Bensoussan, A., Prato, G. Da Delfour, M. , and Mitter, S. K. *Representation and Control of Infinite Dimensional Systems*, Second edition,Birkh\"auser Boston, Inc., Boston, MA, 2007.
5. Liu, W. *Elementary Feedback Stabilization of the Linear Reaction-Convection-Diffusion Equation and the Wave Equation*, Springer 2009.

MS 778 Advanced Fourier Analysis (L3-T1-P0-CH4-CR4)

Unit 1

Basics of Fourier series, Plancherel theorem for FS, harmonic functions in the unit disc, Poisson integrals, Hilbert transform on the circle and its properties.

Unit 2

Fourier transform of L^1, L^2 and L^p , $1<p<2$ functions, Properties of Fourier transform.

Unit 3

Interpolation Theorems: Riesz-Thorin, Marcinkiewicz

Unit 4

Approximations to the identity, Hardy-Littlewood Maximal Function and boundedness, Lebesgue differentiation theorem, Dyadic Maximal function, and its boundedness.

Unit 5

Hilbert transform and its boundedness by Calderon Zygmund theorem

Unit 6

Various method for singular integrals, Calderon-Zygmund theorem for singular integrals and its extension to the vector valued functions

Textbook:

1. Duoandiketxea, J. *Fourier Analysis*, Graduate Studies in Mathematics, Vo. 29, AMS, 2001

References:

1. Stein, E. M. and Shakarchi, R. *Functional Analysis*, Princeton Lect. In AnalysisVol-4, 2011
2. Stein, E. M. and Weiss, G. *Introduction to Fourier Analysis in Euclidean Spaces*, Princeton University Press, Princeton, 1971.
3. Sadosky, C. *Interpolation of Operators and Singular Integrals: An Introduction to Harmonic Analysis* Marcel Dekker, Inc., 1979.
4. Katznelson, Y. *An Introduction to Harmonic Analysis*, Cambridge Mathematical Library, Reprint 2004.
5. Chandrasekharan, K. *Classical Fourier Transform*, Universitext, Springer Verlag, 172, 1989.
6. Grafakos, L. *Classical Fourier Analysis*, Graduate Text in Mathematics, Springer, 249, 2009
7. Grafakos, L. *Modern Fourier Analysis*, Graduate Text in Mathematics, Springer, 250, 2009

MS 779 Fractional Calculus (L3-T1-P0-CH4-CR4)

Unit- 1

Special Functions of Fractional Calculus: Spaces of Integrable, absolutely Continuous and Continuous Functions, Gamma function, Beta functions, Classical Mittag-Leffler Functions, Wright Functions, the popular definitions of fractional derivatives/Integrals in Fractional calculus.

Unit -2

Riemann-Liouville Fractional Derivatives: Riemann-Liouville Fractional Integrals and Fractional Derivatives of constant, polynomial, exponential and trigonometric functions, Laplace Transform of Mittag-Leffler, Riemann Liouville Fractional Integrals and Riemann Liouville Fractional Derivatives, Grunwald-Letnikov fractional derivative.

Unit- 3

Caputo Fractional Derivatives: Caputo Fractional Derivatives of constant, polynomials, exponential and trigonometric functions, Laplace Transform of Caputo Fractional Derivatives, Comparison and relation between Riemann Liouville and Caputo fractional Derivatives, Hilfer derivatives as a generalization of Riemann Liouville and Caputo fractional Derivatives.

Unit -4

Linear Fractional Differential Equations: Solutions of standard fractional differential equations using Laplace Transform, Power series, Mellin Transform, Fourier transform method, Existence and uniqueness theorem as a method of solution, Dependence of a solution on initial conditions.

Unit- 5

Fractional Abstract Evolution Equations: Evolution Equations with Riemann-Liouville Derivative, Evolution Equations with Caputo Derivative, Nonlocal Cauchy Problems for Evolution Equations

Textbook(s):

1. Podlubny, I. *Fractional Differential Equations*, San Diego: Academic Press, 1999.
2. Kilbas, A. A., Srivastava, H. M, and Trujillo, J.J. *Theory and Applications of Fractional Differential Equations*, North Holland Mathematical Studies Vol. 204, Elsevier (North-Holland) Sciences Publishers, Amsterdam, 2006.

Reference Book(s):

1. Miller K. S. and Ross, B. *An Introduction to the Fractional Calculus and Fractional Differential Equations, New York: Wiley,1993.*
2. Zhou Y., *Basic Theory of Fractional Differential Equations*, World Scientific,2014.
3. Samko, S.G., Kilbas, A.A. and Marichev, O.I. *Fractional Integrals and Derivatives: Theory and Applications,* Gordon and Breach Science, 1993.

Course Title: **Theory of Partitions**

Course Code: MS780

Credit: 4 (L2-T2-P0-CH4)

Prerequisites: Elementary Number Theory, Basic Real and Complex Analysis, Combinatorics

Course Contents:

Unit 1: Definition of Partition, Graphical representation of partitions, Infinite products and q-series identities, Generating functions for partitions, Euler Identity, Euler's pentagonal number theorem.

Unit 2: Partition identities, Rogers-Ramanujan functions and identities, Hardy-Ramanujan-Rademacher formula for the partition function.

Unit 3: Ramanujan's partition congruences and their various proofs, Rank and Crank of a partition function.

Unit 4: Modular forms and its applications in partitions. Dedekind's eta-function and modular transformation properties.

Unit 5: Plane-partitions, Regular partitions, t-core partitions, Overpartitions, Generalized Frobenius partitions.

Unit 6: Recent trends in the study of partitions.

Textbooks:

1. Andrews, G. E. *Theory of Partitions*, Cambridge University Press, 1998.
2. Berndt, B. C. *Number Theory in the Spirit of Ramanujan*, AMS, 2006.
3. Murty, M. R. , Dewar, M., and Graves, H. *Problems in the Theory of Modular Forms*, Springer, 2016.

References:

1. Andrews, G. E. *Number Theory*, Dover, 1994.
2. Ono, K. *The Web of Modularity: Arithmetic of the Coefficients of Modular Forms and q-series*, CBMS Regional Conference Series, AMS, 2004.
3. Recent papers from journals such as *Journal of Number Theory, Ramanujan Journal, International Journal of Number Theory, Transactions of the AMS, Discrete Mathematics, Research in Number Theory, Mathematical Notes etc. and archives.*

Course Title: General Theory of Relativity and Gravitational Waves

Course Code : MS 781

Credit: 4(L2-T2-P0-CH4)

Course content

Unit-1:

Brief review of Special Relativity, transition from Special Theory of Relativity to General Theory of Relativity, Principle of equivalence, consequences, the need for a curved space-time. (8 lectures)

Unit-2:

2-dimensional curved spaces example of the sphere, paraboloid etc., principle of covariance and tensors, elementary properties of tensors. Riemannian space, Metric tensor, Parallel Transport, Christoffel symbols of first and second kind, Covariant derivative of Tensors, Geodesic Equation, Riemannian Tensor, Ricci Tensor, Bianchi Identities, Scalar Curvature, Einstein Tensor and their properties, Differential Manifolds. (12 lectures)

Unit-3:

Einstein Field Equations, Energy-momentum stress tensor, the Newtonian limit.
(4 lectures)

Unit-4:

Spherically symmetric solution to Einstein Field Equation in free space and in matter, Schwarzshild line element. Schwarzschild Singularity, Kruskal-Szekeres coordinates.
(8 lectures)

Unit-5:

Linearised theory of gravity, weak field limit, Lorenz gauge and wave solution to Einstein Field Equations, gravitational Waves, Transverse-Traceless gauge, Polarisation properties, detection of gravitational waves, generation of gravitational waves, Landau-Lifschitz quadrupole formula, Energy and momentum of gravitational waves, decay in the orbit of the Hulse-Taylor binary pulsar, detection of GW150914 of binary black holes by LIGO detectors. (14 lectures)

Unit-6:

Main sequence stars, Advanced stages of stellar evolution, Gravitational collapse, white dwarf, neutron star, black hole, Gravitational waves from High Energy Astrophysics phenomena (6 lectures)

Unit-7:

Numerical Relativity, basic idea. 3+1 formation. Difference equation. (8 lectures)

Textbooks:

1. Dhurandhar S. V. and Mitra S.: General Relativity and Gravitational Waves - Essentials of Theory and Practice, Springer 2022.
2. Schutz, B.F.: 7A first course in General Relativity, Cambridge University Press, 3rd ed., 2022.
3. Narlikar J. V.: An introduction to Relativity, Cambridge University Press 2010.
4. Carroll, S., Spacetime and Geometry. Addition-Wesley, 2004