SYLLABUS
&
SCHEME OF EXAMINATION

Ph. D.
in
MATHEMATICAL SCIENCES

DEPARTMENT OF MATHEMATICAL SCIENCES
TEZPUR UNIVERSITY
2007
(Modified: 2011)
### COURSE STRUCTURE

#### Compulsory Courses

<table>
<thead>
<tr>
<th>Course No.</th>
<th>Name</th>
<th>L-T-P</th>
<th>CH</th>
<th>CR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS 701</td>
<td>Research Methodology in Mathematical Sciences-I</td>
<td>0-1-4</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>MS 702</td>
<td>Research Methodology in Mathematical Sciences-II</td>
<td>0-1-4</td>
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</table>

Total Credit = 6

#### Elective Courses

<table>
<thead>
<tr>
<th>Course No.</th>
<th>Name</th>
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<th>CH</th>
<th>CR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS 703</td>
<td>Magnetohydrodynamics</td>
<td>2-1-0</td>
<td>3</td>
<td>3</td>
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<tr>
<td>MS 704</td>
<td>Programming &amp; Numerical Methods</td>
<td>2-0-2</td>
<td>4</td>
<td>3</td>
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<tr>
<td>MS 705</td>
<td>Ramanujan's Theta Functions and Applications to Number Theory</td>
<td>2-1-0</td>
<td>3</td>
<td>3</td>
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<tr>
<td>MS 706</td>
<td>Topological Structures</td>
<td>2-1-0</td>
<td>3</td>
<td>3</td>
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<tr>
<td>MS 707</td>
<td>Fuzzy Sets &amp; Fuzzy Logic</td>
<td>2-1-0</td>
<td>3</td>
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<tr>
<td>MS 708</td>
<td>Algebraic Methods in Operator Theory</td>
<td>2-1-0</td>
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<tr>
<td>MS 709</td>
<td>Probability Measure, Random Variable and Probability Distribution</td>
<td>2-1-0</td>
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<td>MS 710</td>
<td>Operators on Spaces of Analytic Functions</td>
<td>2-1-0</td>
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<tr>
<td>MS 711</td>
<td>Theory of Distributions and Sobolov Spaces</td>
<td>2-1-0</td>
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<td>MS 712</td>
<td>Finite Element Methods (FED) for PDEs</td>
<td>2-1-0</td>
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<td>MS 714</td>
<td>Advanced Matrix Theory</td>
<td>2-1-0</td>
<td>3</td>
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<td>MS 715</td>
<td>Non-Negative matrix Theory</td>
<td>2-1-0</td>
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<tr>
<td>MS 716</td>
<td>Number Fields and Elliptic Curves</td>
<td>2-1-0</td>
<td>3</td>
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</tbody>
</table>

L: Lectures  T: Tutorials  P: Practical  CH: Contact Hours (all per week)  CR: Credit

* A student has to select **any two** of the above elective courses, i.e. **Total Credit** to be completed by a student = **6**
DETAILS OF SYLLABI

Compulsory Course

MS 701: Research Methodology in Mathematical Sciences-I  (L0-T1-P4-CH5-CR-3)

Data Collection and Literature Survey, Application software like, Mathematica/MatLab, Latex & different versions.

MS 702: Research Methodology in Mathematical Sciences  (L0-T1-P4-CH5-CR-3)

State-of-the-art Seminar presentations, Submission of report in the format of a standard research article.

Elective Courses
(Any two are to be chosen by a student)

MS 703 Magnetohydrodynamics  (L2-T1-P0-CH3-CR3)


References:
2. Ferraro, V. C. A. & Plumpton, C., Magnetohydrodynamics, Oxford University Press
3. Pai, S. I., Magnetohydrodynamics, Springer-Verlag

MS 704: Programming and Numerical Methods  (L2-T0-P2-CH4-CR3)

References:

MS 705: Ramanujan's Theta Functions & Applications to Number Theory  
(L2-T1-P0-CH3-CR3)

Ramanujan's general theta-function, Special cases and their relations, q-series and infinite products, Jacobi triple product identity, Schröter's formulae and theta-function identities. Modular equations, Class invariants, Evaluation of class invariants, Explicit values of theta-functions. Ramanujan's continued fractions and explicit values. Applications of theta-functions to the theory of partitions, Ramanujan's famous congruences for the partition function, Rogers-Ramanujan-type functions and partition theoretic interpretations.

References:

MS 706: Topological Structures  
(L2-T1-P0-CH3-CR3)

Basics of point set topology, Uniform structures, uniform continuity, completeness and completions, metrizability of uniform spaces; different uniform structures in topology, Paracompactness, countably paracompact space, weakly and strongly paracompact space, metrization results; Dimension theory, basic properties of three dimension functions ind, Ind and dim, properties of dim, the imbedding theorem.

References:
MS 707: Fuzzy sets and Fuzzy Logic  (L2-T1-P0-CH3-CR3)

Fuzzy sets - basic definitions, alpha-level sets, convex fuzzy sets, basic operations on fuzzy sets, types of fuzzy sets, cartesian products, algebraic products, bounded sum and difference, t-conorms. The extension principle - the Zadeh's extension principle, image and inverse image of fuzzy sets, fuzzy numbers, elements of fuzzy arithmetic. Fuzzy relations and fuzzy graphs, composition of fuzzy relations, min-max composition and its properties, fuzzy equivalence relations, fuzzy graphs. Fuzzy logic, fuzzy propositions, fuzzy quantifiers, linguistic variables, inference from conditional fuzzy propositions, compositional rule of inference. Approximate reasoning - an overview of fuzzy expert systems, fuzzy implications and their selection, multi-conditional approximate reasoning, role of fuzzy relation equation. Rough sets, intuitionistic fuzzy sets and applications

References:

MS 708 Algebraic Methods in Operator Theory  (L2-T1-P0-CH3-CR3)

Fredholm operators, Semi-Fredholm operators, index of a Fredholm (semi-Fredholm) operator, Weighted shifts and their norm and spectral radii; normaloid, convexoid and spectraloid operators, Toeplitz operators.
Transitive, Reductive, Reflexive operator algebras; strictly cyclic and cyclic operator algebras, triangular and quasi triangular operator algebras.
The Hardy Spaces, Beurling’s Theorem and its extensions, The Inner-Outer Factorization of Functions in $H^2$. 

References:

MS 709: Probability Measure, Random Variable and Probability Distribution (L2-T1-P0-CH3-CR3)


References:

MS 710: Operators on Spaces of Analytic Functions (L2-T1-P0-CH3-CR3)

Prerequisite:
1. The basic facts about Holomorphic Functions of one complex variable.
2. The Lebesgue Theory of Measure and Integration.
3. A first course in Functional Analysis, including basics of Operator Theory.
Preliminary Operators on Banach Spaces and Hilbert Spaces, Compact operators, Schatten class operators, Hilbert-Schmidt operators.


A study of Hardy spaces $H^p$ with special reference to the Hardy-Hilbert space $H^2$.

Hankel and Toeplitz operators on the $H^2$ space.

Hankel and Toeplitz operators on the Bergman space, Composition operators on the Hardy and Bergman spaces.

Text books:

Reference books:

MS 711: Theory of Distributions and Sobolov Spaces
(L2-T1-P0-CH3-CR3)

Prerequisite:
1. Advanced Real Analysis
2. Measure Theory
3. Functional Analysis

Definition and basic properties of Sobolev spaces, Approximation by smooth functions, Extension theorems, Inclusion relations and Sobolev’s inequality, Negative norms and Duality, Fractional order Sobolev space, Trace Theorems.
Interpolation, Bounds for the interpolation error, Inverse estimates, Interpolation of non-smooth functions.
Text books:

Reference books:

**MS 714 Advanced Matrix Theory** [L2-T1-P0-CH3-CR3]
Prerequisites: MS 403 Linear Algebra

Review of Linear Algebra, Unitary matrices, Unitary equivalence, Normal matrices, Schur’s theorem, Hermitian and symmetric matrices, variational characterization of eigenvalues, application of variational characterizations, Positive definite matrices, Positive semidefinite matrices, Singular value decomposition, Schur complement,

Text Books:

Reference Books:

**MS 715 Non-Negative Matrix Theory** [L2-T1-P0-CH3-CR3]
Prerequisites: MS 403 Linear Algebra


Text Books:
Reference Books:

MS 712: Finite Element Methods (FEM) for PDEs

Prerequisite:
1. Introduction to Numerical Analysis
2. Sobolev Spaces
3. Partial Differential Equation


FEM for one and two dimensional elliptic problems, Lax-Milgram Theorem, regularity estimates, construction of finite element spaces, error estimates, Aubin-Nitsche duality argument

FEM for parabolic problems, spatially discrete schemes, The backward Euler and Crank Nicolson fully discrete schemes, error analysis

FEM for hyperbolic problems, spatially and fully discrete schemes and error analysis.

Numerical quadrature, Isoparametric finite elements, Numerical examples using Matlab

Text Books:

Reference Books:
Number fields and their rings of integers, Prime decomposition in number rings, The ideal class group and the unit group, Dirichlet's Unit Theorem, Dedekind zeta function and the class number formula, Class Numbers of Quadratic Fields and Cyclotomic fields. Introduction to algebraic curves, singular and non-singular curves, Mordell-Weil group law on elliptic curve, explicit formulas for group law, points of finite order, Nagell-Lutz theorem, Mordell’s theorem, rank of elliptic curves. Elliptic curves over finite fields, Complex multiplication.

**Text Books:**
1. Marcus, D. A. *Number Fields*, 3rd edition (Springer-Verlag, 2009)

**Reference Books:**