Learning Outcomes Based Curriculum

Department of Mathematical Sciences
Tezpur University

M.Sc. in Mathematics

Preamble

Department of Mathematical Sciences, Tezpur University strive to implement LOCF (Learning Outcomes based Curriculum Framework) as suggested by University Grants Commission (UGC) and proposed to be implemented by Tezpur University. The basic structure of the M.Sc. Programme is designed keeping in mind the following facts:

1. The learning outcomes of each paper are designed so that these may help learners to understand the main objectives of studying the course.
2. This will enable learners to select elective papers depending on the individual inclinations and contemporary requirements.
3. The objective is to prepare the students to learn Mathematics leading to M.Sc. Degree and motivate them towards higher learning of mathematics through research.
4. This syllabus in Mathematics is implemented keeping in view the wide applications of Mathematics in Science, Engineering, Social science, Business and a host of other areas.
5. The study of the syllabi will enable the students to be equipped with the state of the art of the subject and will empower them to get jobs in technological and engineering fields as well as in business, education and healthcare sectors.

1. Introduction

The M.Sc. programme in Mathematics consists of 4 semesters comprising 80 credits in all. Besides the prescribed compulsory papers, each student has to opt for at least 4 elective papers during the course of study. The course has been designed to equip the students with theoretical knowledge as well as problem solving techniques. The academic curriculum requires each final year student to undertake a project (4 credits) in any branch of mathematics or related to mathematics to facilitate his/her independent thinking.

2. Qualification descriptors for the graduates

Knowledge & Understanding

i. Demonstrate fundamental systematic knowledge of mathematics and its applications in engineering, science, technology and mathematical sciences and research. It also enhance the subject specific knowledge.

Skills & Techniques

i. Demonstrate educational skills in areas of analysis, geometry, algebra, mechanics, differential equations etc.

Competence

i. Apply knowledge, understanding and skills to identify the difficult/unsolved problems in mathematics and to collect the required information in possible range of sources and try to analyse and evaluate these problems using appropriate methodologies.
ii. Fulfil one's learning requirements in mathematics, drawing from a range of contemporary research works and their applications in diverse areas of mathematical sciences.
iii. Apply one's disciplinary knowledge and skills in mathematics in newer domains and uncharted areas.
iv. Identify challenging problems in mathematics and obtain well-defined solutions.
v. Exhibit subject-specific transferable knowledge in mathematics relevant to job trends and employment opportunities.

3. Graduates Attributes

The graduate attributes in Mathematics are the summation of the expected course learning outcomes. Some of them are stated below.

i. Capability of demonstrating comprehensive knowledge of mathematics and understanding of one or more disciplines of mathematics.

ii. Ability to communicate various concepts of mathematics effectively using examples and their geometrical visualizations.

iii. Ability to use mathematics as a precise language of communication in other branches of human knowledge.

iv. Ability to employ critical thinking in understanding the concepts in every area of mathematics.

v. Ability to analyze the results and apply them in various problems appearing in different branches of mathematics.

vi. Ability to provide new solutions using the domain knowledge of mathematics by framing appropriate questions relating to the concepts in various fields of mathematics.

vii. To know about the advances in various branches of mathematics.

viii. Capability to understand and apply the programming concepts of C to mathematical investigations and problem solving.

ix. Ability to work independently and do in-depth study of various notions of mathematics.

x. Ability to think, acquire knowledge and skills through logical reasoning and to inculcate the habit of self learning.

4. Program Outcomes

M.Sc. in mathematics

PO1. Have good knowledge and exposure to basic and contemporary fields of Mathematics.
PO2. Have excellent domain knowledge in chosen elective subjects leading to research.
PO3. Inculcate abstract mathematical and higher order logical thinking and aptitude in problem solving.
PO4. Develop good computation, programming, data analysis skills and their applications.
PO5. Become competent to succeed in national level competitive examinations for taking up doctoral research and teaching.

5. Programme structure

Programme Name: M.Sc. in mathematics

Total Credits: 80

Structure of the curriculum

Course category		No of courses	Credits per course	Total Credits
I.	Core courses	18	2 to 4	62
II.	Elective courses			
	Department Specific Elective (DSE)	03	04	12
	Open Elective	02	03	06
	Total credits	**23**	**2 to 4**	**80**

6. SEMESTER-WISE SCHEDULE

SEMESTER I

Course type	Course title	Lecture (L)	Tutorial (T)	Practical (P)	Contact Hour (CH)	Credits
Core	MS 401: Abstract Algebra	3	1	0	4	4
	MS 403: Linear Algebra	3	1	0	4	4
	MS 405: Real Analysis	3	1	0	4	4
	MS 411: Computer Programming	3	1	0	4	4
	MS 421: Computer Lab	0	0	1	2	1
	MS 425: Lebesgue Measure and Integration	3	1	0	4	4

SEMESTER II

Course type	Course title	Lecture (L)	Tutorial (T)	Practical (P)	Contact Hour (CH)	Credits
Core	MS 406: Complex Analysis	3	1	0	4	4
	MS 408: Topology	3	1	0	4	4
	MS 414: Theory of Ordinary Differential Equations	3	1	0	4	4
	MS 416: Numerical Analysis[+]	3	1	0	4	4
	MS 424: Computer Lab	0	0	2	4	1
Elective	Open Elective-I[#]	2	1	0	3	3

+ Course for which there is a separate practical unit assigned as Computer Laboratory.

List to be notified by the CoE from time to time.

SEMESTER III

Course type	Course title	Lecture (L)	Tutorial (T)	Practical (P)	Hour (CH)Contact	Credits
Core	MS 507: Partial Differential Equations	3	1	0	4	4
	MS 510: Functional Analysis	3	1	0	4	4
	MS 517:Project	0	2	2	6	4
Elective	DSE-I*	3	1	0	4	4
	Open Elective II#	2	1	0	3	3

SEMESTER IV

Course type	Course title	Lecture (L)	Tutorial (T)	Practical (P)	Hour (CH)Contact	Credits
	MS 501: Classical Mechanics	3	1	0	4	4
	MS 508: Mathematical Methods	3	1	0	4	4
	MS 599: Probability Theory	3	1	0	4	4
Elective						
	DSE-II*	3	1	0	4	4
	DSE-III*	3	1	0	4	4

* are to be chosen from a list of elective courses offered by the department of Mathematical Sciences.

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List of currently offered elective courses:

Course Code	Course Name	L-T-P	CH	CR
MS 538	Theory of Partial Differential Equation	3-1-0	4	4
MS 539	Advanced Numerical Analysis	3-1-0	4	4
MS 541	Fluid Mechanics	3-1-0	4	4
MS 543	Relativity	3-1-0	4	4
MS 549	Graph Theory	3-1-0	4	4
MS 552	Operator Theory-I	3-1-0	4	4
MS 554	Commutative Algebra	3-1-0	4	4
MS 558	General Theory of Relativity	3-1-0	4	4
MS 561	Stochastic processes-I	3-1-0	4	4
MS 565	Fuzzy Sets and Applications-I	3-1-0	4	4
MS 566	Fourier Analysis	3-1-0	4	4
MS 567	Continuum Mechanics	3-1-0	4	4
MS 568	Theory of Distribution and Sobolev Spaces	3-1-0	4	4
MS 569	Coding Theory-I	3-1-0	4	4
MS 570	Coding Theory-II	3-1-0	4	4
MS 572	Operator Theory –II	3-1-0	4	4
MS 573	Analytic Number Theory	3-1-0	4	4
MS 574	Galois Theory	3-1-0	4	4
MS 581	Stochastic Processes –II	3-1-0	4	4
MS 585	Fuzzy Sets and Applications-II	3-1-0	4	4
MS 588	Applied Matrix Theory	3-1-0	4	4
MS 591	Computational Fluid Dynamics	3-1-0	4	4
MS 594	Advanced Topology-I	3-1-0	4	4
MS 596	Advanced Topology-II	3-1-0	4	4

7. Mapping of course with program outcomes (POs)

Course title	PO1	PO2	PO3	PO4	PO5
MS 401: Abstract Algebra	√	√	√	√	√
MS 403: Linear Algebra	√	√	√		√
MS 405: Real Analysis	√	√	√	√	√
MS 411: Computer Programming				√	√
MS 425: Lebesgue Measure and Integration	√	√	√		√
MS 421: Computer Lab				√	√
MS 406: Complex Analysis	√	√	√		√
MS414: Theory of Ordinary Differential Equations	√	√		√	√
MS 408: Topology	√	√	√		√
MS 416: Numerical Analysis+	√	√	√	√	
MS 424: Computer Lab			√	√	
MS 501: Classical Mechanics	√	√		√	√
MS 507: Partial Differential Equations	√	√	√		√
MS 508: Mathematical Methods	√	√	√	√	√
MS 510: Functional Analysis	√	√	√		√
MS 517: Project	√	√	√	√	√
MS 538: Theory of Partial Differential Equations	√	√	√		√
MS 539: Advanced Numerical Analysis	√	√	√	√	√
MS 541: Fluid Mechanics	√	√	√		√
MS 543: Relativity	√	√	√		√
MS 549: Graph Theory	√	√	√		√
MS 552: Operator Theory-I	√	√	√		√
MS 554: Commutative Algebra	√	√	√		√
MS 558: General Theory of Relativity	√	√	√		√
MS 561: Stochastic Process-I	√	√	√	√	√
MS 565: Fuzzy Sets and Applications-I	√	√	√		√
MS 566: Fourier Analysis	√	√	√	√	√
MS 567: Continuum Mechanics	√	√	√		√
MS 568: Theory of Distribution and Sobolev Spaces	√	√	√		√
MS 569: Coding Theory I	√	√	√	√	√
MS 570: Coding Theory -II	√	√	√	√	√
MS 572: Operator Theory -II	√	√	√		√
MS 573: Analytic Number Theory	√	√	√	√	√
MS 574: Galois Theory	√	√	√		√
MS 581: Stochastic Process-II	√	√	√	√	√
MS 585: Fuzzy Sets and Their Applications-II	√	√	√		√
MS 588: Applied Matrix Theory	√	√	√	√	√
MS 591: Computational Fluid Dynamics	√	√	√	√	√
MS 594: Advanced Topology –I	√	√	√		√
MS 596: Advanced Topology –II	√	√	√		√
MS 599: Probability Theory	√	√	√	√	√

8. Evaluation plan:

There shall be minimum two Sessional Tests and two Examinations for each Theory Course, and two Examinations for a Practical Course having L-T-P structure. Details as follows:

Evaluation plan for Theory Courses:

Sessional Test/ Examination		Course Credit ≤ 2		Course Credit ≥ 3		Semester period
Nomenclature	Type	Marks	Duration	Marks	Duration	
Sessional Test-I	Written	20	30 min	25	45 min	Within 5th week
Mid-Semester Examination	Written	30	90 min	40	2 hours min	Within 10th week
Sessional Test-II	Written/ Assignment/Seminar etc.	20	XX	25	XX	Within 14th week
End-Semester Examination	Written	50	2 hours	60	3 hours	Within 18th week

Evaluation plan for Practical Courses:

Examination		L-T-P Structure-wise Marks		Semester period
		L-T-P: 0-0-z	L-T-P: x-y-z	
Nomenclature	Type	Marks	Marks	
Mid-Semester (Practical) Examination	Viva, Report	30	-	Before Mid Semester
End-Semester (Practical) Examination	Practical Examination, Viva, Report	70	50	Before End Semester

9. DETAILED SYLLABUS

MS 401: Abstract Algebra

L 3 T1 P0 CR4

Course outcomes

Towards the end of this course students will learn

1. basic concepts in group theory, cyclic groups, permutation groups, subgroups, normal subgroups and group homomorphisms
2. structures of finite groups, Sylow Theorems, finite Abelian groups, normal series, and solvable groups
3. rings, fields, homomorphisms, embedding theorems, polynomial rings, factorization theory in integral domains, Euclidean domains, and Gaussian domain
4. separable and inseparable extension of fields, finite fields, and elements of Galois theory

Course content

- External direct product of groups, properties of external direct products, internal direct products, fundamental theorem of finite abelian groups and applications.
- Group action, properties of group action, class equation of finite groups, Sylow's theorems, applications of Sylow's theorems.
- Subnornmal, normal series, derived group, solvable groups, composition series, nilpotent groups, Jordan-Holder theorem.
- Word, reduced word, free group, rank of a free group, fundamental theorem of free groups, presentation of groups.
- Polynomial rings, rings of formal power series, embedding theorems, field of fractions.
- Factorization theory in integral domains, PID, UFD and Euclidean domains.
- Field extensions, algebraic and transcendental elements, geometrical constructions, splitting field, finite fields, structure of finite fields, normal, separable and inseparable extension of fields.

Textbook(s):

- Gallian, J. A., *Contemporary Abstract Algebra*, 4th edition, Narosa Publishing house, New Delhi, 2009.
- Dummit, D. S. & Foote, R. M., *Abstract Algebra*, 3rd edition (John Wiley & Sons, Indian reprint, New Delhi, 2011.
- Herstein, I. N., *Topics in Algebra*, 2nd edition (John Wiley & Sons, Indian reprint, New Delhi, 2006.

Suggested readings:

- Fraleigh, J. B. *A First Course in Abstract Algebra*, 7th edition (Pearson Education India, New Delhi, 2008.
- Lang, S. *Algebra*, 3rd edition (Springer India, New Delhi, 2006).

MS 403: Linear Algebra

L 3 T1 P0 CR4

Course outcomes

1. One of the goals of the course is to emphasize the analytic and geometric techniques of Linear Algebra.
2. Understand the beautiful interplay between abstract theory and concrete applications of Linear Algebra.

Course content

- Matrix representation of a linear transformation, Annihilating polynomial of a linear transformation; Elementary Canonical forms: diagonalization and triangulation of linear operators. Gerschgorin's disk theorem.
- Primary Decomposition theorem; rational and Jordan forms.
- Inner product spaces: inner product, Cauchy-Schwarz inequality, Gram-Schmidt orthogonalization process.
- Linear functionals and adjoints; self adjoint, positive definite, normal and unitary operators.
- Linear functionals and adjoints; self adjoint, positive definite, normal and unitary operators; orthogonal projections; spectral theorem for normal operators on a finite dimensional vector space, Singular value decomposition.
- Bilinear forms, Matrices of bilinear forms, Symmetric bilinear forms, Diagonalization of symmetric matrices, positive and quadratic forms, Sylvester's law of inertia.

Textbook(s):

- Stephen H. F., Arnold J. I. and Lawrence E. S., Linear Algebra, 4th edition, Prentice Hall, 2003.
- Halmos, P. R., Finite dimensional vector spaces, Springer Verlag, New York, 1987.
- Hoffman, K. and Kunze, R., Linear Algebra, Prentice Hall, 1984.

Suggested readings:

- Halmos, P. R., Linear Algebra Problem Book, The Mathematical Association of America (MAA), USA, 1995.
- Williams, G., Linear Algebra with Applications, Jones and Burlet Publishers, 2001.

MS 405: Real Analysis

L 3 T1 P0 CR4

Course outcomes

1. Learn some of the basic but fundamental concepts of real analysis
2. Develop the analytical thinking by solving problems
3. Familiarize the process of generalization (real line to any metric space)

Course content

- Sequence of functions, pointwise and uniform convergence, interchange of limits. Functions of bounded variation. Reimann Stieltjes integral. Integration by parts.
- Compactness, Sequential compactness, Bolzano-Weierstrass Property, Totally bounded spaces, compactness and completeness, finite intersection property. Continuous functions on compact spaces. Characterization of complete metric spaces. Arzela Ascoli Theorem.
- Connectedness, intermediate value theorem, Completeness, Bolzano Weierstrass Theorem, nested set theorem. Fixed point theorem. Completion.
- Functions of several variables, directional derivatives, differentiability and total derivative. Jacobians, chain rule, higher order partial derivatives, Taylor’s theorem. Inverse function theorem, Implicit function theorem, extremum problem with constraints, Lagrange’s method of multiplier.

Textbook(s):

- N. L. Carothers. Real Analysis.
- W. Fleming. Functions of several variables.

Suggested readings:

- Apostol, T. M. Mathematical Analysis, Narosa Publishing House, 1985.
- Simmons, G. F. Introduction to Topology and Modern Analysis (Tata McGraw Hill Book Co. Ltd.,1963).

MS 406: Complex Analysis

L 3 T1 P0 CR4

Course outcomes

Students will be

1. acquainted with the theory of functions of a single complex variable.
2. introduced to the theory of residues in evaluating real improper integrals and definite integrals involving sine and cosie functions, locating zeros of functions, and so on.
3. Introduced to conformal mapping and Mobius transformation which have many applications in other fields of science and engineering.

Course content

- Convergence of sequences and series, Absolute and uniform convergence of power series, Integration and differentiation of power series, uniqueness of series representations.
- Taylor series, Zeros of analytic functions, Limit points of Zeros, Singularities and their classification, Behaviour of the function in a neighbourhood of isolated singularities, Laurent's series, Residues, Cauchy Residue Theorem.
- Evaluation of improper integrals and definite integrals involving sines and cosines, integration through a branch cut.
- The winding number, Logarithmic residues and Rouche's theorem, the Argument Principle.
- Mapping by elementary functions, Linear fractional transformations, cross ratios, mappings of the half planes and circles, conformal mapping, Statement of Riemann Mapping Theorem.
- Schwarz Reflection Principle, Analytic continuation, Riemann Surfaces.

Textbook(s):

- Mathews, J. H. and Howell, R. W., *Complex Analysis for Mathematics and Engineering*, 3rd Edition, Narosa, 1998.
- Conway, J. B. *Functions of One Complex Variable*, 2nd Edition, Narosa Publishing House, India, 1994.
- Churchill, R. V. and Brown, J. W. *Complex Variables and Applications*, McGraw-Hill, Education (India) Edition, 2014.

Suggested readings:

- Ahlfors, L. V. *Complex Analysis*, 3rd Edition (McGraw-Hill Publishing Company, New Delhi, 1979.
- Priestly, H.A. *Introduction to Complex Analysis*, 2nd Edition, Cambridge, 2008.
- Gamelin, T. W., *Complex Analysis*, UTM, Springer, 2003.
- Narasimhan, R. and Nievergelt, Y., *Complex Analysis in One Variable*, 2nd Edition, Springer (India), New Delhi, 2004.

MS 408: Topology

L 3 T1 P0 CR4

Course outcomes

On completion of the course

1. students will understand basic structures and constructions of topology.
2. student will have clear understanding on countability, separation axioms and their applications.
3. develop sufficient knowledge on properties of compact and connected spaces and their localized variations.

Course content

- Metric topology, Product and Box topology, Order topology, Quotient spaces
- Countability axioms: First countable spaces, Second countable spaces, separable spaces, Lindelof spaces.
- Separation axioms: Hausdorff, Regular and Normal spaces, Urysohn’s characterization of normality, Urysohn’s metrization theorem, Tietze’s extension theorem, Completely Regular spaces.
- Compactness, limit point compactness, local compactness, one-point compactification.
- Tychonoff's product theorem, Stone-Cech compactification, Baire Spaces, Baire Category Theorem.
- Connectedness, Local connectedness, Path connectedness, Components, Products of connected spaces.

Textbook(s):

- Munkres, J. R. *Topology : A first course* (2/e), Pearson Education, 2000.
- Willard, S., Topology, Dover, 1970.

Suggested readings:

- Joshi, K. D.**,** *Topology*, Wiley-Eastern, 1988.
- Kelley, J. L.**,** *General Topology,* Graduate texts in Math., Springer, 1991.
- Adams C. and Franzosa, R. Introduction to Topology: Pure and Applied, Pearson, 2009.

MS 411: Computer Programming⁺

L 3 T1 P0 CR4

Course outcomes

At the end of this course student will

1. develop programming skills.
2. learn basic techniques of C-language.
3. develop some expertise in developing programs to solve various mathematical problems

Course content

- Revision of fundamentals of C: Data types in C, variables in C, input output statements, constant declaration, arithmetic operators in C, arithmetic expressions, assignment statements, arithmetic assignment operators, increment and decrement operators, type conversions, operator precedence. for loop, while loop, do…while loop, if statement, if…else statement, switch statement, conditional operators. The break statement, the continue statement, the go-to statement.

- Arrays: Arrays, declaration of one dimensional arrays, two dimensional arrays. Structures and Unions: User defined data types, structures, array of structures, unions, enumerated data type.
- Searching and Sorting: Bubble sort, selection sort, insertion sort, linear search and binary search.
- Function in C: Simple functions, passing arguments to functions with return value, call by value, call by reference, overloaded functions, inline functions, default arguments.
- Pointers: Introduction; accessing address of a variable; pointer declaration, initialization, accessing variable through pointer, chain of pointers; pointer expressions, increment and scale factor. Pointers and Arrays. Array of pointers. Pointers as function arguments.
- Files in C: Defining and opening a file, closing a file. Input/Output operations on files.
- Dynamic Memory Allocation and Linked list: Dynamic memory allocation, Malloc, Calloc, Free, Realloc. Concepts of linked list, advantages of linked list, types of linked list. Creating a linked list.

Textbook(s):

- Rajaraman, V. *Fundamentals of Computers,* Prentice Hall of India, New Delhi,2002.
- Balaguruswamy, E. *Programming in ANSI C,* Tata McGraw-Hill, 2004.

Suggested readings:

- Kanetkar, Y. P. *Let us C* (BPB Publication, 2001).
- Venkateshmurthy, M. G. *Programming Techniques through C,* Pearson Education, 2002.

+ *Practical unit for the course MS 411 to be done in the course MS 421 Computer Laboratory*

MS 414: Theory of Ordinary Differential Equations

L 3 T1 P0 CR4

Course outcomes

1. Understand the qualitative behaviour of various initial and boundary value problems for ordinary differential equations which arise in applications.
2. Solve ordinary differential equations by series solution method.
3. Solve systems of differential equations.
4. Draw the phase portraits and study the stability of solutions
5. Solve the solutions of Strum Liouville problems.

Course content

- *Review of fundamentals of ODEs, Some basic mathematical models, direction fields, classification of differential equation, Solutions of some differential equation. 1st order non-linear differential equation.* Existence and Uniqueness problem, Gronwall's inequality, Peanao existence theorem, Picard existence and uniqueness theorem, interval of definition.
- Second order linear differential equation, General solution for homogeneous equation, superposition of solutions, Methods of solution for non-homogeneous problem: undetermined coefficients, variation of parameters.

- n^{th} order differential equation, system of equation, homogeneous system of equation, fundamental matrix, Abel-Liouvelle formula, System of non-homogeneous equations, Stability of linear systems.

- Theory of two point BVP, Greens function, Greens matrix, properties of greens functions, adjoint and self adjoint BVP.

- Strum-Liouvilles problem, Orthogonal functions, eigen values & eigen functions, Completeness of the Eigen functions.

- Orthogonal trajectory of a system of curves on a surface solution of pfaffian differential equations in three variables.

- Stability of linear and non-linear system: Classification of critical points, Lyanpunov *stability.*

Textbooks:

- Boyce, W. E. and DiPrima, R. C. *Elementary Differential Equation and Boundary Value Problems*, 7th Edition, John Wiley & Sons (Asia), 2001.

- Ross, S. L. *Differential Equations*, 3rd edition (Wiley 1984).

Suggested readings:

- Simmons, G. F. *Differential Equations with Applications and Historical Notes,* McGraw Hill 1991.
- Coddington, E. A. *An Introduction to Ordinary Differential Equations,* Prentice- Hall, 1974.
- Farlow, S. J. *An Introduction to Differential Equations and Their Applications* McGraw-Hill International Editions, 1994.

MS 416: Numerical Analysis $^{+}$

L 3 T1 P0 CR4

Course outcomes

1. Towards the end of the course the student would be able to apply the concepts taught in various research problems as well as develop analytical capability in numerical analysis.

Course content

- Definition and sources of errors, Propagation of errors, Error analysis, Sensitivity and conditioning, Stability and accuracy, Floating-point arithmetic and rounding errors.

- Solution of system of linear algebraic equations: Iterative methods- Jacobi, Gauss-Seidel, Successive over-relaxation (SOR), symmetric SOR (SSOR). Numerical solution of non-linear simultaneous equations, Newton's method, General iteration method.

- Review of interpolation, Hermite interpolation. Spline interpolation, B-splines. Special emphasis on cubic spline.

- Approximation of function: The Weierstrass and Taylor theorem, Minimax and least square approximations, Orthogonal polynomials.
- Numerical solution of algebraic and transcendental equations: Methods based on first and second degree equations, rate of convergence. Theory of one point iterative methods and its convergence analysis, multipoint iterative methods. Numerical evaluation of multiple roots.
- Overview of Nwton-Cotes method. Composite integration. Gaussian quadrature, one point, two point and three point formulae. Orthogonal polynomials, Gauss-Legendre, Gauss-Hermite and Gauss-Laguerre quadrature formulae. Romberg integration.
- Solution of ordinary differential equations. Picard method, Euler method, backward Euler method, modified Euler method, Runge-Kutta class of methods.
- Solving problems with C codes.

Textbook(s):

- Atkinson, K.E. *Introduction to Numerical Analysis,* John Wiley, 1989.
- Jain, M.K., Iyengar, S.R.K. and Jain R.K. *Numerical methods for Scientific and Engineering Computation,* 5th edition, New Age International (P) Ltd., New Delhi, 2006.

Suggested readings:

- Sastry, S.S. *Introductory methods of Numerical Analysis*, Prentice Hall of India, New Delhi, 1977.
- Hilderbrand, F. B. *Introduction to Numerical Analysis,* Tata McGraw Hill, New Delhi, 1974.
- Conte, S. D., Boor, Carl de. *Elementary Numerical Analysis - An Algorithmic Approach*, 3rd Edition, McGraw Hill, 1980.

MS 421: Computer Laboratory+

L 0 T0 P1 CR1

+ *Practical unit for the course MS 411 Computer Programming*

Course outcomes

1. At the end of the course student will able to write Computer program to solve some mathematical problem.

MS 424: Computer Laboratory+

L 0 T0 P1 CR1

+ *Practical unit for the course MI 302 Numerical Analysis+*

Course outcomes

1. Student will able to formulate and solve certain mathematical problems numerically.

MS 425: Lebesgue Measure and Integration

L 3 T1 P0 CR4

Course outcomes

1. At the end of the course the student would be able to have a better idea of the theory of integration and contribute to this classical field of knowledge by solving various problems.

Course content

- Algebra of sets, Borel sets, Extended real numbers.
- Lebesgue measure on the Real Line: Lebesgue outer measure, Lebesgue Measurable sets and Lebesgue measure, Non-measurable sets.
- Lebesgue Measurable functions, Simple functions, Littlewood's principles.
- Lebesgue integral of simple functions, Lebesgue integral of bounded functions, Bounded convergence theorem, Comparison of Riemann and Lebesgue integral.
- Lebesgue integral of non-negative functions, Fatou's Lemma, Monotone convergence theorem, Lebesgue general integral, Lebesgue dominated convergence theorem.
- Convex function and Jensen's inequality, *Lp* spaces, Young, Holder and Minkowski inequalities, Completeness of *Lp* spaces, Bounded linear functionals on *Lp* spaces.

Textbook(s):

- Royden, H.L. and Fitzpatrick, P. M., Real Analysis, 4th Edition, Pearson, 2010.
- Barra, G. De. *Measure Theory and Integration,* New Age International(P) Ltd, Publishers, New Delhi 2003.

Suggested readings:

- Rana, I. K. *An Introduction to Measure and Integration*, 2nd edition, Narosa Publishing House India, 2000.
- Halmos, P. R. *Measure Theory,* Springer-Verlag, 1974.
- Jain, P. K. and Gupta, V. P. *Lebesgue Measure and Integration,* New Age International (P) Limited, New Delhi, 1986.

MS 501: Classical Mechanics

L 3 T1 P0 CR4

Course outcomes

On completion of this course students will

1. have a sound idea of the Newtonian, Lagrangian and Hamiltonian dynamics.
2. work out Canonical transformation to classical mechanical equations.
3. write down equations in classical mechanics in terms of the Poisson's Bracket formulation

Course content

- Momentum and kinetic energy, motion about a fixed point, Euler's equation, General equation of motion for a single particle and a system of N number of particles, General Solution.
- Motion of a heavy sphere in a cylinder and a cone, motion under no force, Torque, Poinst's representation of motion.
- Lagrange's equation of motion for holonomic systems, Velocity dependent potential, conservation theorem and symmetric properties, Lagrange's multiplier for holonomic and nonholonomic systems, Lagrange's equation for impulsive motion.
- Hamiltonian of a dynamical system, Hamilton's canonical equation of motion, Cyclic coordinate, The Routhian, Conservation of energy and momentum.
- Lagrange's method for small oscillation, Normal modes, Equations and examples.
- Integral invariants of poincare, Lagrange's and Poisson's brackets and their properties, Equation of Motion and conserved quantities using Poisson's brackets, Infinitesimal contact transformation.
- Euler's equation of calculus of variations, Brachistochrone problem, extremes under constraints, Hamilton's principle for conservative and non-conservative system, Holonomic and non-holonomic system.
- Derivation of Lagrange's and Hamilton's equations from Hamilton's principle of least action, Hamilton Jacobi theory, Hamilton's Principal Function.

Textbook(s):

- Goldstein, H. *Classical Mechanics*, 2nd edition, Narosa Publishing House, New Delhi, 2000.
- Rana, N. C. & Joag ,P. C. *Classical Mechanics*, Tata-McGraw Hill, 1991.

Suggested readings:

- Takwale, R. G. & Puranik, P. S. *Classical Mechanics*, Tata-McGraw Hill, 1979, 41st reprint, 2010.
- Yung-Kuo, L. *Problems and Solutions on Mechanics*, World Scientific, 1994.
- Calkin, M. G. *Lagrangian and Hamiltonian Mechanics*, World Scientific, 1996.
- Landau, L. & Lifshitz, E.M. *Mechanics: Course of Theoretical Physics, Vol. 1,* 3rd edition Pergamon Press, 1976.

MS 507: Partial Differential Equations

L 3 T1 P0 CR4

Course outcomes

It is expected that at the end of the course the student will be able to

1. apply knowledge of partial differential equations wherever needed.
2. solve partial differential equations appearing in diverse context.

Course content

- Linear and nonlinear partial differential equation of the first order. Cauchy's method of characteristics, Compatible systems of first order equations, Charpit's and Jacobi's method.
- Linear PDE with constant coefficients, reducible and irreducible equations. Different methods of solution.
- Second order PDE with variable coefficients. Characteristic curves of second order PDE. Reduction to canonical forms. Solutions of PDE of second order by the method of separation of variables.
- Fourier transform, Laplace transform. Solution of partial differential equation by Laplace and Fourier transform methods.
- Solutions of PDE of second order by the use Riemann’s method. Adjoint operators. Solutions of PDE of second order by the method of integral transforms.
- Elliptic differential equations. Occurrence and detailed study of the Laplace and the Poisson equation. Maximum principle and applications, Green’s functions and properties.
- Parabolic differential equations. Occurrence and detailed study of the heat equation. Maximum principle. Existence and Uniqueness of solutions of IVPs for heat conduction equation. Green’s function for heat equation.
- Hyperbolic differential equations. Occurrence and detailed study of the wave equation. Solution of three dimensional wave equation. Method of decent and Duhamel’s principle. Solutions of equations in bounded domains and uniqueness of solutions.

Textbook(s):

- Sneddon, I.N. Partial Differential Equations, Diver, 2006.
- Rao, K.S. Introduction to partial differential equations (Prentice Hall of India, New Delhi, 2006.

Suggested readings:

- John, F. Partial Differential Equations, 3rd edition, Narosa, 1979.
- Haberman, R. Elementary Applied Partial Differential equations, Prentice-Hall, New Jersey, 1987.
- Willams, W.E. Partial Differential Equations, Oxford University Press, 1980.
- Strauss, W.A. Partial Differential Equations: An Introduction, John Wiley, 1992.
- McOwen, R. Partial Differential Equations Methods and Applications, Prentice Hall, New Jersey, 1996.

MS 508: Mathematical Methods

L 3 T1 P0 CR4

Course outcomes

1. Towards the end of the course the student would be able to solve basic application-oriented problems. Students also would be able to develop problem solving skills useful in research works.

Course content

- Calculus of variations: Linear functionals, minimal functional theorem, general variation of a functional, Euler-Lagrange equation. Variational problems with fixed boundaries. Sufficient conditions for extremum.
- Integral equations: Linear integral equations of the first and second kind of Fredholm and Volterra type, solution by successive substitutions and successive approximations, solution of equations with separable kernels. Fredholm alternative.
- Nonlinear programming: formulation of NLPP, General NLPP, Kuhn-Tucker condition. Saddle point and NLPP. Graphical solutions of NLPP, quadratic programming. Wolfe's modified simplex method, Beale's method.
- Game theory: Two-person zero-sum games, maximum criterion, dominance rules, mixed strategies, mini-max theorem, solutions of 2x2 and 2xm games.

Textbooks:

- Watson G. N. *A Treatise on the Theory of Bessel Functions,* Cambridge University Press, 1944.
- Brown J. W. and Churchill, R. *Fourier Series and Boundary Value Problems,* McGraw Hill, 1993).
- Roach, G. F. *Green's Functions, Cambridge University Press*, 1995.
- Swarup, K., *Gupta*, P.K., Mohan, M., *Operations Research,* Sultan Chand & Sons, 2007.

Suggested readings:

- Gupta, A, S. *Calculus of Variations with Applications,* Prentice Hall of India, New Delhi 2003.
- Mikhlin, S. G. *Integral equations*, The MacMillan Company, New york, 1964.

MS 510: Functional Analysis

L 3 T1 P0 CR4

Course outcomes

It is expected that at the completion of the course students will

1. be illuminated by abstract approach to analysis and the strong theory which is developed here helps to bring out the essence of a problem by clearing out unnecessary details, hence giving a unified approach to apparently unrelated topics

2. understand major links between mathematics and its applications.
3. able to apply the various concepts to solve numerical problems.
4. get sufficient background to understand the more advanced concepts to undertake further research.

Course content

- Recap pre-requisite topics: Sets and relations, Linear spaces and linear maps, Metric spaces and continuous functions.
- Introduce normed linear spaces with examples. Properties of nls, Riesz lemma, particular study of finite dimensional normed linear spaces, Discuss the interplay between linear structure and metric structure.
- Define stronger, weaker and equivalent norms. Continuity and boundedness of linear maps, introduce complete normed linear spaces with examples. Function spaces and operator norm, bounded linear functional. Definition of Schauder basis. Dual spaces.
- Hahn Banach separation and extension theorems. Applications of HBT.
- Refer to the Ascoli-Arzela theorem and the definitions of uniform continuity, uniform boundedness, equicontinuity of a family of functions. Uniform boundedness theorem.
- Closed graph theorem, open mapping theorem, bounded inverse theorem. Examples and applications of above theorems.
- Inner product and Hilbert spaces, Bessel's inequality, Riesz-Fisher theorem, orthonormal basis. Fourier expansion and relation to orthonormal basis, Parseval formula, Separable Hilbert spaces.
- Approximations, projection theorem, Riesz representation theorem. Hilbert adjoint operator, normal, self adjoint and unitary operators.

Textbooks:

- Limaye B. V. *Functional Analysis*, Wiley Eastern Ltd., New Delhi, 1989.
- Kreyszig E. *Introductory Functional Analysis with Applications*, John Wiley and Sons, New York, 1978.

Suggested readings:

- Rudin W. *Functional Analysis*, McGraw Hill, 2000.
- Yosida K. *Functional Analysis*, Springer, 1995.
- MacCluer B. *Elementary Functional Analysis*, GTM 253, AMS, 2009.
- Siddiqi, A. H., Ahmad K and Manchanda P., *Introduction to Functional Analysis with Applications*, Real World Education Publishers, New Delhi, 2014.

MS 517: Project

L 0 T2 P2 CR4

Course outcomes

At the end of the course student will

1. able to compile existing work
2. learn to prepare review report using LaTex
3. develop skill of presention

MS 538: Theory of Partial Differential Equations

L 3 T1 P0 CR4

Course outcomes

On completion of this course student will

1. understand the fundamental concepts of partial differential equations.
2. able to work out problems in abstract spaces.
3. create the basic knowledge required for research in partial differential equations.

Course content

- Overview of PDE. Laplace equation, mean-value fomulas, strong maximum principle. Heat and wave equations, uniqueness by energy methods.
- Theory of distributions: test functions, distributions, generalized derivatives, Sobolev Spaces, imbedding theorems, Rellich-Kondrasov theorem, trace theory.
- Elliptic Boundary Value Problems: abstract variational problems, Lax-Milgram Lemma, weak solutions, regularity result, maximum principles, eigen value problems.
- Semigroup Theory and Applications: exponential map, C0-semigroups, Hille-Yosida and Lummer-Phillips theorems, applications to heat and wave equations.

Textbooks:

- Kesavan, S., *Topics in Functional Analysis*, New Age International (P) Ltd. 1989, reprint 2003.
- Evans, L.C., *Partial Differential Equations,* AMS, Providence, 1998.

Suggested readings:

- John, F. , *Partial Differential Equations,* 3rd ed., (Narosa Publ. Co., New Delhi, 1979.
- Gilbarg, D. and Trudinger, N. , Elliptic *Partial Differential Equations of Second Order,* Springer-Verlag, Berlin Heidelberg, 2001.
- Jost, J. , *Partial Differential Equations,* Springer-Verlag New York, 2002.
- Renardy, M. and Rogers, R.C. , *An Introduction to Partial Differential Equations,* 2nd ed., Springer Verlag International Edition, New York, 2004.

MS 539: Advanced Numerical Analysis

L 3 T1 P0 CR4

Course outcomes

1. It is expected that at the end of the course the student will be able to understand fundamental concepts of mathematics in different branches mathematics to be offered in the subsequent semesters. This topic will act as the foundations.

Course content

- Finite Difference method: Explicit and Implicit schemes, consistency, stability and convergence, Lax equivalence theorem. Numerical solutions of elliptic, parabolic and hyperbolic partial differential equations.
- Optimization: Problem formulation, single variable optimization, multi variable optimization.
- Krylov subspace methods, Conjugate-Gradient (CG), BiConjugate-Gradient (BiCG), BiCG Stabilised (BiCGStab), Generalised Minimum Residual (GMRES). Preconditioning Techniques, parallel implementations.
- Approximate method of solution: Galerkin method, properties of Galerkin aproximations, Petrov-Galerkin method, Generalised Galerkin method.
- Review of Sobolev spaces. Weak solution of elliptic boundary value problem, regularity of weak solutions, maximum principle. Finite Element method: Definition and properties. Element types triangular, rectangular, quadrilateral. Application of finite element method for second order problems, one and two dimensional problems. Isoparametric finite element, non-conformal finite element. Mixed finite element.

Textbooks:

- Watkins, D. S. *Fundamental of Matrix Computations*, 2nd edition Wiley- Interscience, 2002.
- Smith, G. D. Numerical *Solution of Partial Differential Equations: Finite Difference Methods,* 3rd edition (Oxford University Press, 1986)
- Reddy, J. N. *An Introduction to the Finite Element Method*, 3rd Edition (McGraw Hill India, 2006.

Suggested readings:

- Trefethen, L. N and Bau, David *Numerical Linear Algebra,* SIAM, 1997.
- Hoffman, Joe D. *Numerical Methods for Engineers and Scientist*, 2nd edition, Mc-Graw Hill 2004.
- Ciarlet, P. G. *The Finite Element Method for Elliptic Problems,* North Holland,1978.
- Johnson, C.*Numerical Solution of Partial Differential Equations by the Finite Element Method,* Cambridge University Press, 1987.

MS 541: Fluid Mechanics

L 3 T1 P0 CR4

Course outcomes

1. It is expected that at the end of the course the student will be able to understand fundamental concepts of fluid mechanics. Subsequently students should be in a position to apply the knowledge in diverse context of modelling. This topic will also help students in clearing UPSC as fluid mechanics is in the syllabus of UPSC.

Course content

- Lagrarian and Eulerian methods, Velocity and acceleration, Particle path, Stream lines, Streak lines velocity potential, Steady and unsteady flows.
- Conservation of mass and momentum, Equation of continuity. Equations of motion of a fluid, Pressure at a point in fluid at rest, Pressure at a point in a moving fluid, Euler's equation of motion, Bernoulli's equation.
- Total energy, Circulation, Boundary surface, Impulsive motion. Irrotational motion, Potential flow, Green's theorem, application of Green's theorem in fluid mechanics, Kinetic energy of liquid, Uniqueness theorem. Vorticity vector.
- Motion in two dimensions, Singularities of flow, Source, Sink, Doublets, Rectilinear vortices. Complex variable method for two-dimensional problems, Complex potentials for various singularities, Circle theorem, Blasius theorem, Theory of images and its applications to various singularities.
- Three dimensional flow, Irrotational motion, Weiss's theorem and its applications.
- Viscous flow, Vorticity dynamics, Vorticity equation, Reynolds number, Stress and strain analysis, Navier-Stokes equation, Boundary layer Equations.

Textbooks:

- Munson, B.R.,Young, D.F. & Okiishi, T.H. *Fundamentals of Fluid Mechanics*, 6th ed., John Wiley & Sons, 2009.
- White, Frank M. *Fluid Mechanics*, Mc-Graw Hill, 2005.

Suggested readings:

- Batchelor, G. K. *An Introduction to Fluid Dynamics*, Cambridge University P,1993.
- Panton, R. L. *Incompressible Flow*, John Wiley & Sons, 2005.
- Schlichting, H. *Boundary Layer Theory*, Mc-Graw Hill, 2005.
- Chorlton, F. *Textbook of Fluid Mechanics*, C. B. S. Publishers, Delhi, 1985.
- Besant, W. H & Ramsey, A. *A Treatise on Hydro-mechanics*, ELBS, 1990.

MS 543: Relativity

L 3 T1 P0 CR4

Course outcomes

On completion of this course, a student will

1. develop a reasonable idea of Tensor analysis and their applications
2. grasp the elements of special theory of relativity and write down equations in Four-vector formulation
3. appreciate the basics of General Theory of Relativity and its applications to astrophysics and cosmology.

Course content

- Postulates of Special Theory of Relativity, Lorentz Transformation and consequences, Minkowski Diagram, Four dimensional spacetime continuum, Four vector formulation, Elementary properties of Tensors.
- Rules of Combination of Tensors, Tensor Algebra, Inner and Outer Product Quotient Rule, Metric tensor and Conjugate Metric Tensors, Raising and lowering properties, Christofell symbols of first and second kind, Transformation properties of Christofell symbols.
- Tensor Analysis, Covariant Derivative of Tensors, Gradient, Divergence and Curl of Tensor, Geodesic Equation, Riemannian Tensor, Ricci Tensor, Bianchi Identities, Einstein Tensor.
- Transition of Special to General Theory of Relativity, Principle of covariance, Equivalence Principle and consequences, Stress Energy Tensor, Einstein Field Equations and Newtonian limit.
- Spherically symmetric solution to Einstein Field Equation in free space and in matter, Schwarzshild line element. Schwarzschild Black Holes.
- Equation of Planetary Orbits, Crucial tests of General Theory of Relativity, Advance of Perihelion, Gravitational bending of light and Gravitational Redshift.
- Cosmology: Large scale structure of Universe, Galactic Densities and the darkness of the Night Sky, Galactic Number Counts, Olber's paradox, Cosmological principles, Relativistic Universe and models.
- Einstein and de-Sitter models of static universe, Dynamical Universe, Comoving time, Red Shifts and Horizons, Friedmann-Robertson-Walker line element, Open and Closed Universe, Hubbles law, Early Universe.

Textbooks:

- Narlikar, J.V. *An Introduction to Cosmology*, 3rd edition, Cambridge University Press, 2002.
- Landau and Lifshitz *Classical Theory of Fields*, Pergamon Press, 1975.

Suggested readings:

- Dirac, P. A. M. *General Theory of Relativity*, Prentice Hall of India, reprinted, 2001.

- Weinberg, S. *Gravitation and Cosmology,* John Wiley & Sons, 1972.
- Kenyon, I. R. *General Relativity,* Oxford University Press, 1991.

MS 549: Graph Theory

L 3 T1 P0 CR4

Course outcomes

At the end of the course students are expected to

1. apply theories and concepts to test and validate intuition and independent mathematical thinking in problem solving.
2. improve the proof writing skills.
3. use graph theory as a modelling tool.
4. apply graph theory based tools in solving practical problems.

Course content

- Preliminaries: Graphs, subgraphs, Isomorphism, degree, degree sequence, operations on graphs.
- Walk, Trail, Path, Cycle, circuit, Connected graphs, component, distance between vertices, Bipartite graph, eccentricity, radius, diameter.
- Tree, Bridge, Center of a tree, Forest, Spanning tree.
- Cut-vertices, Block, vertex-connectivity, edge-connectivity, Eulerian graph and its properties, Hamiltonian graph and its Properties.
- Planarity: Basic Concepts, Plane Graphs, Interior face, exterior face, Euler Identity, Maximal Planar graph.
- Coloring: vertex coloring, chromatic number, The Four Color Theorem, independence number, Brook's theorem, edge Coloring, edge chromatic number, The Five color Theorem.
- Digraph, oriented graph, indgree, outdegree, strong digraph, tournament, transitive tournament.

Textbooks:

- G. Chartrand and P. Zhang, *A First Course in Graph Theory*, Dover Publication, NewYork, 2012.
- J. A. Bondy, U.S. R. Murthy, *Graph Theory with Applications*, London: Macmillan Press, 1976.

Suggested readings:

- D. B. West, *Introduction to Graph Theory*, 2nd Edition, Pearson Education, 2015.
- R, J. Wilson, *Introduction to Graph Theory,* 4th Edition, Longman, England, 1996.
- F. Harary, *Graph Theory,* Narosa Publishing House, New Delhi, 2001.

MS 552: Operator Theory I

L 3 T1 P0 CR4

Course outcomes

1. This is an introductory course in Operator Theory. It will introduce the student to terms, concepts and results for bounded linear operators which are commonly used in this particular area of Mathematics.
2. It will also introduce the students which are relevant to current research and prepare the student to persue such a career.

Course content

- Uniform, strong and weak convergences.
- Compact linear operators on normed linear spaces; the ideal of compact operators; the separability of the Range and spectral properties of a compact operator; operator equations involving compact operators.
- Bounded operators on Hilbert spaces; adjoint operators; normal, unitary and self adjoint operators; spectral properties of bounded self adjoint linear operators.
- Positive operators and their square root; projection operators; spectral representation of a bounded self adjoint linear operator.
- Spectral measure.
- Spectral theorem for bounded normal operators.

Textbooks:

- Conway, J. B. *A course in Operator Theory,* AMS., GSM Vol. 21, 1999.
- Kreyszig, E. *Introductory functional analysis with applications,* John Wiley and Sons, 1978.

Suggested readings:

- Halmos, P. R. *Introduction to Hilbert spaces and theory of spectral multiplicity,* Chelsea Publishing Co., New York, 1957.
- Abramovich, Y. A. and Aliprantis, C. D. *An Invitation to Operator Theory,* AMS, GSM Vol. 50, 2002.

MS554: Commutative Algebra

L 3 T1 P0 CR4

Course outcomes

1. After completion of this course, students are expected to go deeper into the concepts of commutative algebra and apply these concepts to other fields of science and technology. Also this course is a very good platform for research in Algebraic Number Theory, Algebraic Topology, Algebraic Geometry etc.

Course content

- Ideals in commutative rings, operations on ideals, extension and contraction of ideals, Nilradical and Jacobson radicals, prime spectrum of commutative rings.
- Localization of commutative rings and their basic properties.
- Noetherian and Artinian rings, examples.
- Integral extensions, Dedekind domains.
- Hilbert's Nullstellensatz, Noether's normalisation, valuation rings.
- Modules: Elementary properties of modules, Quotient modules, module homomorphisms, Isomorphism theorems, Generation of modules, Direct sum of modules, finitely generated modules, free modules.
- Tensor product of modules and properties, Exact sequences, projective and injective modules.
- Modules over Principal Ideal Domain.

Textbooks:

- McDonalds, I. G. & Atiyah, M. F. *Introduction to Commutative Algebra,* Levant Books, Kolkata, 2007.
- Dummit, D. S. & Foote, R. M. *Abstract Algebra,* Wiley-India, New Delhi, 2011.

Suggested readings:

- Sharp, R. Y. *Step in Commutative Algebra,* Cambridge University Press, Cambridge, 2000.
- Lang, S. *Algebra,* Springer, GTM Vol. 211, New Delhi, 2006.

MS 558 General Theory of Relativity

L 3 T1 P0 CR4

Course outcomes

On completion of this course student will

1. grasp the idea of Tensor analysis
2. idea of General Theory of Relativity -mathematical Approach
3. idea of General Theory of Relativity -application to stellar Astrophysics and Cosmology

Course content

- Elementary properties of Tensors. Rules of Combination of Tensors, Tensor Algebra, Inner and Outer Product Quotient Rule, Riemannian space, Metric tensor and Conjugate Metric Tensors, Raising and lowering properties.
- Parallel Transport, Christofell symbols of first and second kind, Tranformation properties of Christofell symbols, Covariant Derivative of Tensors, Gradient, Divergence and Curl of Tensors.
- Geodesic Equation, Riemannian Tensor, Ricci Tensor, Bianchi Identities, Scalar Curvature, Einstein Tensor and their properties, Differential Manifolds.

- Transition of Special to General Theory of Relativity, Principle of covariance, Equivalence Principle and consequences, Stress Energy Tensor, Einstein Field Equations and Newtonian limit.
- Spherically symmetric solution to Einstein Field Equation in free space and in matter, Schwarzshild line element. Schwarzschild Singularity, Eddington-Finkelstein co-ordinates, Kruskal-Szekeres co-ordinates.
- Linearised theory of gravity, weak field limit, Hilbert gauge and wave solution to Einstein Field Equations, Gravitational Waves, Polarisation properties, emission of gravitational waves.
- Crucial tests of General Theory of Relativity, Advance of Perihelion, Gravitational bending of light, Gravitational Redshift, Shapiro delay.
- Large scale structure of Universe, Cosmological principles, Einstein and de-Sitter models of static universe, Dynamical Universe, Comoving time, Red Shifts and Horizons, Friedmann-Robertson-Walker line element, Hubbles law, Elements of Quasi-Steady State Cosmology.

Textbooks:

- Narlikar, J.V. *An Introduction to Cosmology*, 3rd edition, Cambridge University Press, 2002.
- Adler, R., Bazin M. & Schiffer, M., *Introduction to General Relativity*, McGraw Hill, 1975.

Suggested readings:

- Landau and Lifshitz, *Classical Theory of Fields*, Pergamon Press, 1975.
- Dirac, P. A. M. *General Theory of Relativity*, Prentice Hall of India (reprinted), 2001.
- Weinberg, S. *Gravitation and Cosmology*, John Wiley & Sons, 1972.
- Kenyon, I. R. *General Relativity*, Oxford University Press, 1991.
- Misner, C., Thorne, K.S. & Wheeler, J.A. *Gravitation*, W.H. Freeman, 1973.

MS 561 Stochastic Processes I

L 3 T1 P0 CR4

Course outcomes

Student will

1. understand the propertise of simple random walk & solve problems using combinatorial methods & generating functions
2. understand discrete Markov Chain, its application and solve problem regading classification of states, periodicity properties
3. understand Poisson process and solve problems
4. understand Renewal theory basics

Course content

- Simple (one dimensional) random walk. (To follow the chapter on simple random walk in Feller (1996) Vol. I)
- Discrete Markov chains: transition probability matrix, classifications of states and chains.
- Introduction to Poisson Processes.
- Introduction to Renewal processes.

Textbooks:

- Feller, W. *An Introduction to Probability Theory and its Applications*, Vol. I, Wiley,1966.
- Medhi, J. *Stochastic Processes*, Wiley Eastern Ltd., New Delhi, 1994.

Suggested readings:

- Bhattacharya, R. and Waymire, E. C. *Stochastic processes with applications*, SIAM, 1990.

MS 565 Fuzzy Sets and Applications-I

L 3 T1 P0 CR4

Course outcomes

Towards the end of the course, student will

1. understand the motivation for the creation of fuzzy sets
2. able to describe the basic mathematical structure and operations of fuzzy sets

Course content

- Fuzzy sets - basic definitions, α-level sets, convex fuzzy sets.
- Basic operations on fuzzy sets, types of fuzzy sets.
- Cartesian products, algebraic products, bounded sum and difference, t-norms and t-conorms. Fuzzy sets in contrast of probability theory.
- The extension principle - the Zadeh's extension principle, image and inverse image of fuzzy sets.
- Fuzzy numbers, elements of fuzzy arithmetic.
- Fuzzy relations and fuzzy graphs, composition of fuzzy relations, min-max composition and its properties, fuzzy equivalence relations, fuzzy relational equations, fuzzy graphs.

Textbooks:

- Klir, G. J. and Yuan, B. *Fuzzy Sets and Fuzzy Logic : Theory and Applications*, Prentice Hall of India, New Delhi, 1997.

Suggested readings:

- Zimmermann, H. J. *Fuzzy set theory and its Applications,* Allied publishers Ltd., New Delhi, 1991.

MS566: Fourier Analysis

L 3 T1 P0 CR4

Prerequisite: MS 510

Course outcomes

At the end of this course student will

1. understand the basics of the Fourier series and its applications to the boundary value problems.
2. understand the convergence of the Fourier series.
3. compute the Fourier transform for various class of functions.
4. use the Fourier transform and its properties for singular integrals and various applications to partial differential equations.

Course content

- Fourier series: Orthogonal systems, Trigonometric system, Orthogonal polynomials, Genesis of the Fourier series.
- Convergence of Fourier series: Fejer mean and Cesaro mean, Convergence of the Fourier series, Fejer theorem, Uniqueness and convergence, Approximate identity, Fourier series of continuous and smooth functions.
- L^2 theory of Fourier series: Inversion formula and the Parseval identity.
- Fourier transforms, the Schwartz space,Plancherel formula, Maximal function and distributions, Tempered distribution,, Fourier analysis and filters. Bessel functions.
- Fourier analysis and complex function theory: Paley Wiener's theorem, Tauberian theorem, Dirichlet problem.,Classical Hardy spaces F and M. Reisz theorem.

Textbooks: (max 2)

- Katznelson, Y. *An Introduction to Harmonic Analysis,* Dover, New York, 1976.

Suggested readings:

- Dym, I.H. and Mc Kean, H.P. *Fourier Series and Integrals,* Academic Press,1985.
- Folland G. B. *Fourier Analysis and Applications,* Brooks/Cole Mathematics Series, 1972.
- Korner, T. *Fourier Analysis,* Cambridge, 1989.
- Rudin, W. *Functional Analysis,* Tata Mc. Graw Hill, 1974.
- Elias M. S. and Shakarchi, R. *Fourier Analysis An Introduction,* Princeton University Press, Princeton, 2004.

MS 567: Continuum Mechanics

L 3 T1 P0 CR4

Course outcomes

At the end of this course student will have a fair knowledge on

1. essential mathematical concepts of continum mechanics, especially stress and strain.
2. fundamental laws of continum mechanics, their derivations.
3. analytical study of equation of elasticity and fluids.

Course content

- Analysis of Strain: Lagrangian and Eulerian finite strain tensor. Geometrical interpretation of the components of atrain. Strain quadric of Cauchy. Principall strains and invariants. General infinitesimal deformation. Saint-Venant's equations of compatibility.
- Analysis of stress: Stress tensor. Equations of equilibrium. Transformation of co-ordinates. Stress quadric of Cauchy. Principal stress and invariants. Maximum normal and shear stresses. Two dimensional problems-Plane stress. Generalised plane stress. Airy stress function.
- Fundamental laws of continuum mechanics: Continuity equation, Equation of motion, Moment of momentum principle, Second law of thermodynamics. The Clausius-Duhem inequality.
- Equations of Elasticity. Generalosed Hooks's law. Homogeneous isotropic media, Strain energy function and its connection with Hooke's law. Four basic elastic constants-Young's modulus, Poisson's ratio, modulus of rigidity, bulk modulus. Uniqueness of solution. Saint-Vernant's principle.
- Fluids: Classification, constitutive equations, energy equation, dissipation of energy.

Textbooks:

- Mase, G.E. *Schaum's Outline of Continuum Mechanics*, Schaum's Outline series, Mc- Graw Hill, 1990.
- Chatterjee, R.   *Mathematical Theory of Continuum Mechanics*, Narosa, 1999.

Suggested readings:

- Truesdell, C. *The elements of continuum Mechanics (*Springer-Verlag, 2000).

MS 568: Theory of Distribution and Sobolev Spaces

L 3 T1 P0 CR4

Course outcomes

1. Students will be well versed in the theoretical aspects of solutions of pdes and apply the same in solving basic pdes.

Course content

- Test Function and distribution: Definition, operations with distributions, convolution of distributions, Fourier transform of tempered distributions.
- Sobolev spaces: generalized derivatives, Definition and properties, extension theorem, imbedding and completeness theorem, fractional order Sobolev spaces, trace theory.
- Application to Elliptic Problems: Weak solution of elliptic boundary value problem (BVP), regularity of weak solutions, maximum principle, eigenvalue problems, fixed point theorems and their application in semi-linear elliptic BVP.

Textbooks:

- Adams, R.A. *Sobolev Spaces,* Academic Press, 1975.
- Kesavan, S. *Topics in Functional Analysis and Applications,* Wiley Eastern Ltd., New Delhi, 1989.
- Strihartz, Robert S. *A guide to Distribution Theory and Fourier Transforms, Studies in Advanced Mathematics,* CRC Press,USA,1994.

Suggested readings:

- Oden, J.T. and Reddy, J.N. *An Introduction to Mathematical Theory of Finite Elements,* Wiley Interscience, 1976.
- Brennan, K. E. and Scott., R. *The Mathematical Theory of Finite Element Methods,* Springer-Verlag, Berlin, 1994.
- Lieb. Elliot H. and Loss, M. *Analysis,* Narosa Publishing House, New Delhi, 1997.
- Rudin, W. *Functional Analysis,* Tata Mc-Graw Hill, 1974.

MS 569: Coding Theory I

L 3 T1 P0 CR4

Course outcomes

1. Towards the end of the course the student would be able to have a better idea about coding theory and how efficient codes are constructed and applied in different areas of practical interest.

Course content

- Communication channel, Introduction to coding theory, types of codes, ISBN code, Barcodes, Digital codes, Group Theory, Vector spaces over arbitrary fields with examples, linear block codes, Dual codes, Distance of block codes, Standard array, Syndrome decoding and Decoding by coset leaders.

- Error-correction and detection capabilties of linear block codes. Singleton bound, Greismer bound, Plotkin bound, Hamming sphere packing bound, Varshamov-Gilbert-Sacks bound.
- Weight Enumerators and the MacWilliams Theorem, Type of errors, Burst errors, Bounds for burst-error detecting and correcting codes.
- Some Interesting Block Codes and Their Properties: Perect codes, Hamming codes, Golay codes, Hadamard codes, Product codes, Reed-Muller codes, Maximum–Distance Separable (MDS) codes.

Textbooks:

- W.W. Peterson and E.J. Weldon, Jr., *Error-Correcting Codes*, M.I.T. Press, Cambridge, Massachusetts, 1972.
- Torleiv Klove, *Codes for error Dectection, Series on Coding Theory and Cryptology*, vol. 2, World Scientific Publishing Co. Pte. Ltd., 2007.

Suggested readings:

- J.H. Van Lint, *Introduction to Coding theory, Graduate Texts in Mathematics*, 86, Springer, 1998.
- Raymond Hill, *A First Course in Coding Theory*, Oxford University Press, 1990.
- A. Neubauer, J. Freudenberger, V. Kuhn, *Coding Theory: Algorithms, Architectures and Applications*, John Wiley & Sons Ltd, England, 2007.
- L.R. Vermani, *Elements of Algebraic Coding*, Chapman and Hall, 1996.
- W. C. Huffman and V. Pless, *Fundamentals of Error-Correcting Codes,* Cambridge University Press, Cambridge, Reprint, 2010.
- Shu Lin and Daniel J. Costello, *Error Control Coding-Fundamentals and Applications*, Pearson Education India, 2011.

MS 570: Coding Theory II

L 3 T1 P0 CR4

Prerequisite: MS 569

Course outcomes

1. Towards the end of the course the student would be able to handle and manipulate difficult type of codes which can be implemented in diverse

Course content

- Zero of polynomials, Algebraic extension of a field, Galios field, Primitive elements, Minimum polynomials, order, Multiplcative group of a Galios fields, structure of finite fields.

- Error detection with cyclic codes, Error-correction procedure for cyclic codes, Shortended cyclic codes, Pseudo cyclic codes. Code symmetry, Invariance of codes under transitive group of permutations.
- BCH codes, Minimum distance and BCH Bounds, Decoding of BCH codes, Reed-Solomon codes.
- Tree codes, Convolutional codes, Description of linear tree and convolutional codes by matrices, distance for convolutional codes, Maximum likelihood decoding of Convolutional codes, Vitorbi decoding algorithm.

Textbooks:

- W.W. Peterson and E.J. Weldon, Jr., *Error-Correcting Codes*, M.I.T. Press, Cambridge, Massachusetts, 1972.
- Shu Lin and Daniel J. Costello, *Error Control Coding-Fundamentals and Applications*, Pearson Education India, 2011.

Suggested readings:

- Man Young Rhee, *Error Correcting Coding Theory*, McGraw-Hill Publishing, 1989.
- Robert H. Morelos-Zaragoza, *The art of Error Correcting Codes*, 2nd Edition, John Wiley & Sons Ltd, England, 2006.
- A. Neubauer, J. Freudenberger, V. Kuhn, *Coding Theory: Algorithms, Architectures and Applications*, John Wiley & Sons Ltd, England, 2007.
- L.R. Vermani, *Elements of Algebraic Coding*, Chapman and Hall, 1996.
- Jiri Adamek, *Foundations of Coding: Theory and Applications of Error-Correcting Codes with an Introduction to Cryptography and Information Theory*, John Wiley & Sons, USA, 1991.
- W. C. Huffman and V. Pless, *Fundamentals of Error-Correcting Codes*, Cambridge University Press, Cambridge, Reprint, 2010.

MS 572: Operator Theory II

L 3 T1 P0 CR4

Course outcomes

This course will introduce the students

1. special classes of bounded linear operator and study why each of them is important and significant.
2. a parallel study of unbounded linear operator is also done to give the student a complete perspective.

Course content

- Functional calculus and spectral mapping theorem for analytic functions; Riesz decomposition theorem.
- Numerical range of an operator; spectral radius.
- Subnormal and hyponormal operators.

- Partial isometries; polar decomposition.
- Unbounded linear operators and their Hilbert adjoint operators; symmetric and self adjoint linear operators; spectral properties of self adjoint linear operators; closed linear operators; closable operators and their closures, multiplication operator and differentiation operator.
- Spectral representation of unitary and self adjoint linear operators.

Textbooks:

- Conway, J. B. *A course in Operator Theory* (AMS., GSM Vol. 21, 1999).
- Kreyszig, E. *Introductory functional analysis with applications*, John Wiley and Sons, 1978.
- Conway, J. B. *A course in Functional Analysis*, Springer Verlag, New York,1985.

Suggested readings:

- Halmos, P. R. *Introduction to Hilbert spaces and theory of spectral multiplicity,* Chelsea Publishing Co., New York, 1957.
- Abramovich, Y. A. and Aliprantis, C. D. *An Invitation to Operator Theory,* AMS., GSM Vol. 50, 2002.

MS 573: Analytic Number Theory

L 3 T1 P0 CR4

Course outcomes

1. Towards the end of the course the student would be able to understand many of the current literature in Analytic Number Theory and Theory of partitions.

Course content

- Arithmetical functions and Dirichlet multiplication, averages of arithmetical functions.
- Elementary theorems on the distribution of primes, the prime number theorem, Chebyshev's functions and their relations.
- Dirichlet's theorem for primes of the form $4n-1$ and $4n+1$, distribution of primes in arithmetic progressions.
- Quadratic residues and quadratic reciprocity law, applications of the reciprocity law, Gauss sums.
- Dirichlet series, Euler products, Riemann zeta function and Dirichlet L-functions.
- Introduction to partitions, geometric representation, generating functions, Euler's Pentagonal number theorem, Jacobi triple product identity, recursion formula for $p(n)$.
- Partition identities of Ramanujan.

Textbooks:

- Apostol, T. M. *Introduction to Analytic Number Theory*, Springer International Student Edition, Narosa Publishing House, New Delhi, 1993.

- Hardy, G.H. and Wright, E. M. *An Introduction to the Theory of Numbers*, 4th Edition, Oxford University Press, 1960.

Suggested readings:

- Niven, I. and Zuckerman, H. *An Introduction to the Theory of Numbers*, 5th Edition, Wiley Eastern, New Delhi, 2000.
- Andrews, G.E. *Number Theory,* Hindustan Publishing Corporation, New Delhi, 1992.

MS 574: Galois Theory

L 3 T1 P0 CR4

Course outcomes

1. After completion of this course, students are expected to go deeper into the concepts of Galois theory and apply these concepts to other branches of science and technology. Also this course is a very good platform for research in Algebraic Number Theory, Algebraic Geometry etc.

Course content

- Field extensions: Algebraic, normal and separable extensions of field.
- Splitting fields.
- Automorphisms of extensions, the fundamental theorem of Galois theory.
- Finite fields.
- Primitive elements, norm and trace, cyclotomic fields, cyclic extension.
- Solution of polynomial equations by radicals, Kummer theory.

Textbooks:

- Morandi, P. *Field and Galois Theory*, GTM Vol. 167, Springer-Verlag, 1996.
- Lang, S. *Algebra,* Springer Verlag, Indian Edition, 2008.
- Dummit & Foote *Abstract Algebra,* John Wiley & Sons., 2005.

Suggested readings:

- Cohn, P. M *Algebra*, Vols. I & Vol. II, John Wiley & Sons, 1985 and 1988.

MS 581: Stochastic Process –II

L 3 T1 P0 CR4

Course outcomes

1. After completion students will understand the basics of Brownian motion process, branching process, weiner process and some basic Queing systems

Course content

- Branching processes- Properties of generating functions of Branching processes, Probability of Extinction, Distribution of the total number of progeny.
- Galton-Watson process. Introduction Brownian motion process.
- Wiener process, first passage time distribution for Wiener process, Ornstein-Ulhenbech process.
- Queueing systems, Single server queueing models (M/M/1/µ, M/M/1/k, M/M/µ/µ, etc.)

Textbooks:

- W. Feller *An Introduction to Probability Theory and its Applications*, II, Wiley, 1998.
- Bhattacharyya, R. and Waymire, E. C. *Stochastic processes with applications,* SIAM, 1990.

Suggested readings:

- Medhi, J. *Stochastic Processes,* Wiley Eastern Ltd., New Delhi, 1994.

MS 585: Fuzzy Sets and Their Applications-II

L 3 T1 P0 CR4

Course outcomes

On completion of this course student will

1. able to describe the fuzzy logic and the approximate reasoning based on the fuzzy logic
2. understand the application of fuzzy logic based system for decision making processes

Course content

- Fuzzy logic, fuzzy propositions, fuzzy quantifiers, linguistic variables, inference from conditional fuzzy propositions, compositional rule of inference.
- Approximate reasoning - an overview of fuzzy expert systems, fuzzy implications and their selection, multi-conditional approximate reasoning, role of fuzzy relation equation.
- An introduction to fuzzy control - fuzzy controllers, fuzzy rule base, fuzzy inference engine.
- Fuzzification, defuzzification and the various defuzzification methods.

- Decision making in fuzzy environment - individual decision making, multi-person decision making, multi-criteria decision making, multistage decision making, fuzzy ranking methods.
- Fuzzy logic as a tool in soft computing.

Textbooks:

- Klir, G. J. and Yuan, B. *Fuzzy Sets and Fuzzy Logic : Theory and Applications,* Prentice Hall of India, New Delhi, 1997.

Suggested readings:

- Zimmermann, H. J. *Fuzzy set theory and its Applications,* Allied publishers Ltd., New Delhi, 1991.

MS588: Applied Matrix Theory

L 3 T1 P0 CR4

Course outcomes

On completion of this course student will

1. able to apply the concept of matrix theory to other field
2. be Equipped with various tools of matrix theory

Course content

- Review of basic linear algebra.
- Canonical factorization, Q-forms.
- Courant-Fischer minmax & related theorems. Perron-Frobenius theory. Matrix-stability.
- Inequalities, g-inverse (A^-, A^m, A^+).
- Direct, iterative, projection and rotation methods for solving linear systems & eigenvalue problems.
- Applications

Textbooks:

- Datta, K. B. *Matrix and Linear Algebra,* PHI, 1991.
- Watkins, D. S. *Fundamentals of Matrix Computation,* Wiley, 1991.
- Golub, G. H. and Loan, C. F. Van. *Matrix Computation,* John Hopkin U. Press, Baltimore, 1996.

Suggested readings:

- Stewart, G. W. *Introduction to Matrix Computations,* Academic Press, 1973.

MS591: Computational Fluid Dynamics

L 3 T1 P0 CR4

Course outcomes

On completion of this course student will be familiar with

1. basic tools used in finite difference and finite volume methods.
2. development of numerical methods for solution of partial differential equations frequently arising in the field of fluid mechanics.
3. theoretical Fluid Dynamics and basic concept of computational Fluid Dynamics in terms of comprehensive theoretical study and computational aspects

Course content

- Basic equations of Fluid Dynamics. Analytical Aspects of PDE. Iterative methods Stationary Methods. Krylov subspace methods.
- Stationary Convection diffusion equations, Non-stationary convection diffusion equations. Conservation laws. Incompressible plane flows, Stream function and vorticity equations, Conservative form and normalizing systems, Method for solving vorticity transport equation, Basic Conservative property.
- Finite volume and finite difference methods. Convergence and stability analysis, Explicit and implicit methods, Stream function equation and boundary conditions, Solution for primitive variables. Finite volume method, Application to Euler equations, Upwind difference scheme, Viscous flow solutions, Total variation diminishing schemes, Godunov-type schemes.
- Simple CFD Techniques, Lax-Wendroff Technique, Mac Cormack's Techniques, Staggered grid, SIMPLE Algorithm. Numerical Solutions of Navier-Stokes equations on collocated and on staggered grids.

Textbooks:

- Chung, T.J. *Computational fluid Dynamics*, Cambridge University Press, 2005.
- Fletcher, C. A. J. *Computational Techniques for Fluid Dynamics, Volume 1 & 2,* Springer Verlag, 1992.

Suggested readings:

- Chow, C. Y. *Introduction to Computational Fluid Dynamics,* John Wiley, 1979.
- Holt, M. *Numerical Methods in Fluid Mechanics,* Springer Verlag, 1977.
- Wirz, H. J. and Smolderen, J. J. *Numerical Methods in Fluid Dynamics* Hemisphere, 1978.
- Anderson, J. D. *Computational Fluid Dynamics,* Mc-Graw Hill, 1995.
- Anderson, D. A., Tannehill, J. C. and Pletcher, R. H. *Computational Fluid Dynamics and Heat Transfer,* McGraw Hill, 1984.

MS594: Advanced Topology –I

L 3 T1 P0 CR4

Course outcomes

On completion of this course student will

1. understand the process of generalization of sequences in topological spaces to deal with their inadequacy.
2. deal with the metrizability problem and have knowledge of some of the metrization results, including application of paracompact spaces.
3. develop basic ideas of homotopy theory and algebraic topology.

Course content

- Nets and filters, convergenge in terms of nets and filters, ultrafilters and compactness.
- Urysohn's Lemma, Tietze Extension theorem.
- Theories of metrization, Urysohn metrization theorem.
- Paracompactness, characterisation in regular spaces, metrization based on paracompactness.
- Nagata-Smirnov theorem, Stone’s theorem, Smirnov’s metrization theorem.
- Homotopy and the fundamental group, computation of the fundamental group of the circle.

Textbooks:

- Joshi, K. D. *Topology* (Wiley-Eastern, 1988).
- Munkres, J. R. *Topology: A first course* (2/e) (Prentice-Hall, 2000).

Suggested readings:

- Kelley, J. L. *General Topology,* Graduate texts in Mathematics, Vol. 27, Springer,1991.
- Willard, S. *General Topology,* Addision-Wesley, Reading, 1970.

MS596: Advanced Topology – II

L 3 T1 P0 CR4

Course outcomes

On completion of this course student will

1. understand the process of creating uniform structures and know the properties and structures of uniform spaces.
2. know the basic structure, properties and applications of topological groups.
3. construct and compare various topologies on spaces of functions.

Course content

- Uniformities, uniform continuity, product uniformities, metrisation.
- Completeness and compactness in uniform spaces.
- Topological groups, subgroups, quotient groups, homogeneous spaces, product groups.
- Uniform structures in topological groups, complete groups, completion of topological groups.
- Function spaces, point-wise convergence, uniform convergence, compact-open topology, k-spaces, equi-continuity, Ascoli theorem.

Textbooks:

- Joshi, K. D. *Topology*, Wiley-Eastern, 1988.
- Munkres, J. R. *Topology: A first course* (2/e), Prentice-Hall, 2000.

Suggested readings:

- Kelley, J. L. *General Topology* (Graduate texts in Mathematics), Vol. 27, Springer, 1991.
- Willard, S. *General Topology,* Addision-Wesley, Reading, 1970.
- Engelking, R. *General Topology,* Polish Scientific Publishers, Warshaw, 1977.
- Bourbaki, N. *Elements of Mathematics: General Topology*, Vols. I & II, Springer-Verlag, 1988.

MS 599: Probability Theory

L 3 T1 P0 CR4

Course outcomes

At the end of the course students

1. have a good concept of the basic of probability, sampling theory and the methods of testing hypothesis and apply the concepts in various problems.

Course content

- Measurable space, Measure and its properties, finite and sigma-finite measures, Axiomatic definition of Probability, Measurable functions , definition of Random Variable.
- Measure induced by a measurable function, definition of Probability distribution and distribution function, properties of distribution function and classification of distributions.
- Some basic theorems Integration theory(integration of measurable functions w. r. t. an arbitrary measure): Fatou's lemma, Monotone Convergence theorem, Dominated convergence Theorem. Definition of Mathematical Expectation of a random variable and its properties. Moments and moment inequalities.
- Convergence of a sequence of random variables (Weak convergence or convergence in probability, almost sure convergence and convergence in Law). Borel-Cantelli-lemma, Weak and Strong law of large numbers, Central limit theorem.

Textbooks:

- Feller, W. *An Introduction to Probability Theory and its Applications*, Vol. II, Wiley, 1966.

- Chow, Y. and Teicher, H. *Probability Theory , Independence, Interchangeability, Martingales,* 3rd Edition, Springer, 1997.

Suggested readings:

- Ash, R. B. *Probability and Measure Theory, Second Edition,* Harcourt/Academic Press, 2000.