TUTORIAL

MS 105: Mathematics II

Unit-3: Complex Integration

Note: All Notations and symbols have their usual meaning.

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(1) Find $\int_C \bar{z} dz$ where C is given by $x = 3t,\ y = t^2,\ -1 \leq t \leq 4$.

(2) Find $\int_C \frac{1}{z^3} dz$ where C is the first quadrant of the circle $|z| = 4$.

(3) If a function f is analytic in a simply connected domain D and C is any contour in D, then show that $\int_c f(z)dz$ is independent of the path C.

(4) Show that $f(z) = \sin z$ is not a bounded function, where z is a complex variable.

(5) Expand $f(z) = \frac{1}{1-z}$ in a Taylor series with center $z_0 = 2i$.

(6) Evaluate $\oint\limits_C \frac{z+4}{z^2+2z+5} dz$, where C is the circle $|z + 1| = 1$.

(7) Evaluate $\oint\limits_C \frac{z^2-z+1}{z-1} dz$, where C is the circle $|z| = \frac{1}{2}$.

(8) Evaluate $\oint\limits_C \frac{dz}{\sqrt{z^2+2z+2}}$, around the unit circle $|z| = 1$ starting with $z = 1$, assuming the integrand to be positive for this value.

(9) Evaluate

(a) $\oint\limits_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$

(b) $\oint\limits_C \frac{\exp 2z}{(z+1)^4} dz$

where C is the circle $|z| = 3$.

(10) Evaluate $\frac{1}{2\pi i} \oint\limits_C \frac{e^z}{(z-2)} dz$, if C is:

(a) the circle $|z| = 3$ and

(b) the circle $|z| = 1$.

(11) Evaluate $\oint\limits_C \frac{e^{-z}}{z+1} dz$, where C is the circle $|z| = \frac{1}{2}$.

(12) Evaluate $\oint\limits_C \frac{3z^2+z}{z^2-1} dz$, where C is the circle $|z - 1| = 1$.

(13) Evaluate $\frac{1}{2\pi i} \oint\limits_C \frac{\exp zt}{(z^2+1)^2} dz$, if $t > 0$ and the circle $|z| = 3$.

(14) Evaluate $\oint\limits_C \frac{\cos^2 tz}{z^3} dz$, if $t > 0$ and the circle $|z| = 1$.

(15) Show that $\oint\limits_C \frac{dz}{z+1} = 2\pi i$, if the circle $|z| = 2$.

(16) Find the Laurent series about the indicated singularities for each of the following functions. Name the singularities in each case.

(a) $\frac{e^{2z}}{(z-1)^3}$; $z = 1$

(b) $(z-3)\sin\frac{1}{z+2}$; $z = -2$

(c) $\frac{z-\sin z}{z^3}$; $z = 0$

(d) $\frac{z}{(z+1)(z+2)}$; $z = -2, z = -1$

(e) $\frac{1}{z^2(z-3)^2}$; $z = 3, z = 0$

(17) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series valid for: a) $0 < |z| < 3$, b) $|z| > 3$, c) $0 < |z+1| < 2$, d) $|z| < 1$.

(18) Expand $f(z) = \frac{1}{(z-3)}$ in a Laurent series valid for: a) $|z| < 3$, b) $|z| > 3$.

(19) Expand $f(z) = \frac{z}{(z-1)(z-3)}$ in a Laurent series valid for: a) $|z| < 1$, b) $1 < |z| < 2$, c) $|z| > 2$, d) $|z-1| > 1$, e) $0 < |z-2| < 1$.

(20) Expand $f(z) = \frac{1}{z(z-2)}$ in a Laurent series valid for: a) $0|z| < 2$, b) $|z| > 2$.

(21) Expand $f(z) = \frac{1}{(z-2)^2}$ in a Laurent series valid for: a) $0|z| < 2$, b) $|z| > 2$.

(22) Expand $f(z) = \frac{z}{(z^2+1)}$ valid for $|z-3| > 2$.

(23) Expand each of the following function in a Laurent series about $z = 0$, naming the type of the singularities in each case. a) $\frac{(1-\cos z)}{z}$, b) $\frac{e^{z^2}}{z^3}$, c) $z^2 e^{-z^4}$.

(24) Determind and classify all the singularities of the functions: a) $\frac{1}{(2\sin z-1)^2}$, b) $\frac{z}{e^{\frac{1}{z}}-1}$, c) $\cos(z^2 - z^{-2})$.

(25) Find the residues of a) $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ and b) $\frac{e^z}{\sin^2 z}$ at all its poles in the finite plane.

(26) For each of the following functions, determine the poles and residues at the poles: a) $\frac{2z+1}{z^2-z-2}$, b) $(\frac{z+1}{z-1})^2$, c) $\frac{\sin z}{z^2}$, d) $\cot z$.

(27) Evaluate $\oint_C e^{-\frac{1}{z}} \sin(\frac{1}{z}) dz$ around the circle C defined by $|z| = 1$.

(28) Find $\oint_C e^{\frac{1}{z^k}} dz$, where $k \in \mathbf{Z}$ ans C is a simple closed contour enclosing the origin.

(29) Evaluate $\oint_C \frac{e^z-1}{z(z-1)(z-i)^2} dz$ where C is the circle $|z| = 2$.