MS 103 MATHEMATICS II

HOME ASSIGNMENT: ROOT FINDING, NUMERICAL INTEGRATION, INTEGRAL TRANSFORM

1. Perform three iterations of the bisection method to obtain the smallest positive root of the equation $x^3 - 5x + 1 = 0$.

2. A root of the equation $xe^x - 1 = 0$ lies in the interval (0.5, 0.1). Determine this root correct to three decimal places using (i) secant method, (ii) regula-falsi method.

3. Using Newton-Raphson method, derive formula to find (i) $1/N$, (ii) $N^{1/q}$, $N > 0$, q integer. Hence find $1/18$, $18^{1/3}$ to four decimal places.

4. Find an interval of unit length which contains the smallest positive root of the equation $x^3 - 5x - 1 = 0$. Hence determine the number of iterations required by the bisection method so that $|error| < 10^{-3}$. Also find the root.

5. Find an interval of unit length which contains the smallest positive root of the equation $e^x - 2x^2 = 0$. Use (i) secant method and (ii) regula-falsi method to find the root. Also by taking the mid-point of this interval as initial approximation, perform two iterations of the Newton-Raphson methods. Compare convergence in each case.

6. An iteration method is defined by $x_{n+1} = \frac{x_n}{2a}(3a - x_n^2)$, $a > 0$, $n = 0, 1, ...$ Find the quantity to which the method converges. Carry out three iterations by using a suitably chosen initial approximation.

7. Evaluate the integral $\displaystyle\int_0^{\pi/2} e^{-x} \cos x dx$, using trapezoidal rule with $h = \pi/2$ and $h = \pi/4$.

8. Find the minimum number of intervals required to evaluate $\displaystyle\int_0^1 \ln(1 + x)dx$ using Simpson's rule with an accuracy of 10^{-6}.

9. Find λ such that the quadrature formula

$$\int_0^1 \frac{f(x)}{\sqrt{x}} dx \doteq Af(0) + Bf(\lambda) + Cf(1)$$

may be exact for polynomials of degree 3.

10. Using the following data.

x	1	2	3	4
$f(x)$	0.3679	0.1353	0.0498	0.0183

and Trapezoidal rule with $n = 1$ and $n = 3$, determine an approximate value of $\displaystyle\int_1^4 f(x)dx$.

11. Find Laplace transform of the following functions.
   (a) $t^2 e^t$.
   (b) $(\cos t + \sin t)^2$.
   (c) $\sinh 2t + \cosh 2t$.
   (d) $e^t \sin t$.
   (e) $f(t) = t^{5/2}$.
   (f) $f(t) = \begin{cases} \cos t, & 0 \leq t < \pi \\ 0, & t \geq \pi \end{cases}$.

12. Find inverse Laplace transform of the following functions.
   (a) $\frac{s}{s^2-4}$.
   (b) $\frac{s^2+2s+5}{(s-1)(s-2)(s-3)}$.
   (c) $\frac{s+3}{(s-1)(s+2)}$.

13. Solve following initial value problems by using Laplace transform

    (a) $y'' + 2y' - 3y = 3$, $y(0) = 4$, $y'(0) = -7$.

    (b) $y'' - 5y' + 4y = e^{2t}$, $y(0) = 19/12$, $y'(0) = 8/3$.

14. Find the Fourier transform of the following

    (a) $f(t) = \begin{cases} \cos t, & -l \leq t \leq l \\ 0, & |t| > 1 \end{cases}$.

    (b) $f(t) = \begin{cases} -e^{\alpha t}, & t < 0 \\ e^{\alpha t}, & t > 0 \end{cases}$, $\alpha > 0$.

    (c) $f(t) = u_0(t-3)e^{-4t}$.

    (d) $f(t) = 1/(1+t^2)$.

15. Find inverse Fourier transform of the following

    (a) $\frac{e^{-i\omega}}{2(1+i\omega)}$.

    (b) $\frac{e^{-2i\omega}}{2+3i\omega}$.

    (c) $\frac{1}{a^4+\omega^4}$, $a > 0$.

16. Solve following differential equations by using Fourier transform

    (a) $y' - 4y = u_0(t)e^{-4t}$, $-\infty < t < \infty$.

    (b) $y'' + 5y' + 4y = \delta(t-2)$.

17. Determine Z transform of $\{f_n\}$.

    (a) $\{f_n\} = e^{-3n}$.

    (b) $\{f_n\} = \begin{cases} 2, & n = 0, 2, 4, ... \\ 1, & n = 1, 3, 5, ... \end{cases}$.

18. Find inverse Z transform of $F(Z) =$

    (a) $\frac{z}{z+2}$.

    (b) $e^{-2/z}$.

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