MS 103 MATHEMATICS II
HOME ASSIGNMENT: INTERPOLATION

1. Prove the following:
   (a) $\Delta\left(\frac{f_i}{g_i}\right) = \frac{g_i \Delta f_i - f_i \Delta g_i}{g_i g_{i+1}}$.
   (b) $\Delta(f_i g_i) = f_i \Delta g_i + g_{i+1} \Delta f_i$.
   (c) $\sum_{k=0}^{n} \Delta^2 f_k = \Delta f_{n+1} - \Delta f_0$.
   (d) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$.
   (e) $\Delta - \nabla = -\Delta\nabla$.
2. Using $e^{0.82} = 2.270500$ and $e^{0.83} = 2.293319$ find an approximate value of $e^{0.826}$ by using Lagrange interpolation. Obtain a bound on the truncation error.
3. Determine the step size h to be used in the tabulation of $f(x) = \sin x$ in the interval $[1, 3]$ so that the linear interpolation will be correct to four decimal places.
4. Show that the truncation error of quadratic interpolation in an equidistant table is bounded by $\left(\frac{h^3}{9\sqrt{3}}\right) \max |f'''(\xi)|$.
5. Given $f(1.0) = 0.7651977$, $f(1.3) = 0.6200860$, $f(1.6) = 0.4554022$, $f(1.9) = 0.2818186$, $f(2.2) = 0.1103623$. Approximate $f(1.5)$ by various Lagrange polynomials and compare.
6. Given

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

   interpolate at $x = 3.0$ and $x = 5.5$.
7. Fit a cubic through the four points of the table $f(3.2) = 22.0$, $f(2.7) = 17.8$, $f(1.0) = 14.2$, $f(4.8) = 38.3$ and use it to find the interpolated value for $x = 3.0$.
8. For the data

x	-4	-2	0	2	4	6
$f(x)$	-139	-21	1	23	141	451

   construct forward and backward difference tables. Using the corresponding interpolation show that the interpolating polynomial is same.
9. Let $n \geq 1$ and assume $f(x)$ is n times continuously differentiable on some interval $[\alpha, \beta]$. Let $x_0, x_1, ..., x_n$ be $n+1$ distinct numbers in $[\alpha, \beta]$. Then show that

$$f[x_0, x_1, ..., x_n] = \frac{1}{n!} f^{(n)}(\xi)$$

   for some unknown point $\min\{x_0, x_1, ..., x_n\} < \xi < \max\{x_0, x_1, ..., x_n\}$.
10. If $f(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$ is a polynomial of degree n then show that its n-th difference is $a_n n! h^n$.
11. Find $y(8)$ given $y(1) = 24$, $y(3) = 120$, $y(5) = 336$, $y(7) = 720$.
12. Use an appropriate central difference formula to find y when $x = 5.96$, from the data $f(5.85) = 3.46$, $f(5.90) = 8.22$, $f(5.95) = 9.64$, $f(6.00) = 6.00$, $f(6.05) = 2.86$. Explain the reason why you have selected the formula you have used.
13. Use an appropriate central difference interpolation formula to compute $f(5.6)$ from the data given below

x	3	4	5	6	7	8
$f(x)$	6.28	8.92	16.5	12.62	7.35	5.37

14. Use the Lagrange and Newton divided difference formulas to calculate $f(3)$ from the following table.

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

15. The discrete set of X, Y is as given below.

X	1.0	2.0	3.0	4.0	5.0
Y	5.0	24.0	63.0	128.0	225.0

Determine Y at $X = 3.3$ by using Stirling formula.

16. Estimate Y at $X = 0.61$ from the following table by Gauss forward interpolation formula.

X	0.2	0.4	0.6	0.8
Y	0.7	0.65	0.7667	0.925

17. Apply Gauss forward formula to calculate Y at $X = 4.2$ from the following table.

X	2.0	3.0	4.0	5.0	6.0
Y	-3.0	-5.0	1.0	21.0	61.0

18. Employ Gauss forward interpolation formula to calculate Y at $X = 52.50$ from the following table.

X	45.0	50.0	55.0	60.0
Y	0.8509	-0.2624	-0.9998	-0.3048

19. Using Stirling interpolation formula calculate Y at $X = 1.85$ from the following table.

X	1.4	1.6	1.8	2.0	2.2
Y	0.3365	0.4700	0.5878	0.6931	0.7885

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