Assignment

MI/D 224

Topic: Numerical Differentiation

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1. The following data for the function $f(x) = x^4$ is given.

x :	0.4	0.6	0.8
$f(x)$:	0.0256	0.1296	0.4096

   Find $f'(0.8)$ and $f''(0.8)$ using quadratic interpolation. Obtain the bound on the truncation errors.

2. A differentiation rule of the form

$$f'(x_0) = a_0 f_0 + a_1 f_1 + a_2 f_2, (x_k = x_0 + kh)$$

   is given. Find the values of a_0, a_1, a_2 so that the rule is exact for $f \in P_2$. Find the error term.

3. Prove the following formulae which give derivatives in terms of differences.
   $(a) y' = \frac{dy}{dx} = \frac{1}{h}\left(\Delta y - \frac{1}{2}\Delta^2 y + \frac{1}{3}\Delta^3 y - ...\right)$
   $(b) y' = \frac{dy}{dx} = \frac{1}{h}\left(\delta y - \frac{1}{24}\delta^3 y + \frac{3}{640}\delta^5 y - ...\right)$

4. Using the following data, find $f'(5)$:

x :	0	2	3	4	7	9
$f(x)$:	4	26	58	112	466	922

5. Derive the formulae for the first derivative of $y = f(x)$ of $O(h^2)$ using (i) forward difference approximations, (ii) backward difference appoximations, (iii) central difference approximations.

6. Consider the four point formula

$$f'(x_2) = \frac{1}{6h}[-2f(x_1) - 3f(x_2) + 6f(x_3) - f(x_4)] + TE + RE$$

   where $x_j = x_0 + jh, j = 1, 2, 3, 4$ and TE, RE are respectively the truncation error and round-off error. (i) Determine the form of TE and RE, (ii) Obtain the optimum step length h satisfying the criterion $|TE| = |RE|$, (iii) Determine the total error.

7. The formula

$$f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)]$$

is suitable for approximating $f'(x_0)$ when x_0 is the first x- value in a table.
(i) State the truncation error as a power of h.
(ii) Derive the Richadrson extrapolation formula when the step lengths $(a)h, \frac{h}{2}, \frac{h}{4}, (b)h, \frac{h}{3}, \frac{h}{9}$ are used.

8. Derive the approximation formula

$$f'(x) = \frac{1}{2h}[4f(x + h) - 3f(x) - f(x + 2h)]$$

Show that its error term is of the form $\frac{1}{3}h^2 f'''(\xi)$.

9. Use the following data, find $f'(6.0)$, error $= O(h)$ and $f''(6.3)$, error $= O(h^2)$

x :	6.0	6.1	6.2	6.3	6.4
$f(x)$:	0.1750	− 0.1998	− 0.2223	− 0.2442	− 0.2596

Text Book: Numerical Methods (For Scientific and Engineering Computation) by M K Jain, S R K Iyengar, R K Jain.